The Numerical Solution of Flow around a Rotating Circular Cylinder in a Uniform Shear Flow

by

Fumio YOSHINO, Tsutomu HAYASHI, Ryoji WAKA
Tatsuo HAYASHI

Department of Mechanical Engineering

(Received September 3, 1985)

The Navier-Stokes equations were numerically solved on the flow around a rotating circular cylinder in a uniform shear flow for various rotating speeds and shear parameters at the Reynolds number of 80. The aerodynamic forces on the cylinder, stream lines and vorticity distributions around it were determined together with the range of rotating speeds and shear parameters where the largest lift oscillations occur.

1. Introduction

The investigations on the fluid dynamic force acting on a body in a low Reynolds number flow are becoming more and more important in relation to the separation of dust from a flow or the dust collection.[1] The authors first calculated the fluid dynamic force acting on a circular cylinder rotating around its axis in a uniform flow. The results of calculation were presented in the former report[2] (" The formor report " means the reference (2) hereafter.). In this paper, the authors present the results of calculation of the fluid dynamic force acting on the rotating circular cylinder in a uniform shear flow instead of the uniform flow.

2. Fundamental Equations and Method of Calculation

The details of the basic equations, the boundary conditions and the method of calculation were described in the former report,[2] so that they are not repeated here. A few points, however, had better be reminded so as to help easy understanding.

The flow is two-dimensional and incompressible. The basic equations are the vorticity transport equation and the relation of the stream function to vorticity. The boundary conditions are the no-slip condition on the surface of the cylinder and the uniform shear flow infinitely far from the cylinder such as

$$u_\infty = U_c\left(1 + \varepsilon \frac{y}{a}\right) , \quad v_\infty = 0 \qquad (1)$$

where U_c is the velocity at infinitely far upstream and on the straight line through the center of the cylinder, ε the dimensionless vorticity of the uniform shear flow, a the radius of the cylinder and y the coordinate perpendicular to U_c.

The basic equations and the boundary conditions were converted to the finite difference equations.[2] The initial value is the inviscid solution of the flow around a still cylinder in a uniform shear flow.[3]

3. Results and Discussion

The rotational speed ratio V_0 is changed from 0 to -2, i.e., range of unsteady flow in the uniform flow, and the velocity gradient ε is changed from 0 to 0.2 though the results of calculation by Tamura et al.[4] are referred to for $\varepsilon > 0.015$ when $V_0 = 0$.

Figures 1 and 2 show the time-dependent C_l and C_d, respectively, where C_l and C_d are the lift and drag coefficients. The mean values $\bar{C}_l$ and $\bar{C}_d$ are the averages of C_l and C_d, respectively, in the region considered

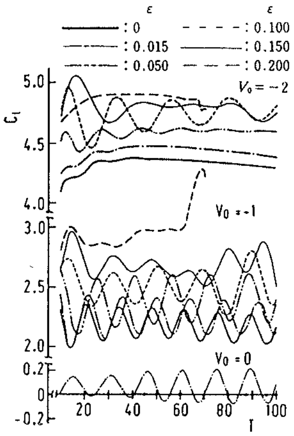

Fig.1 Variation of lift coefficient with time

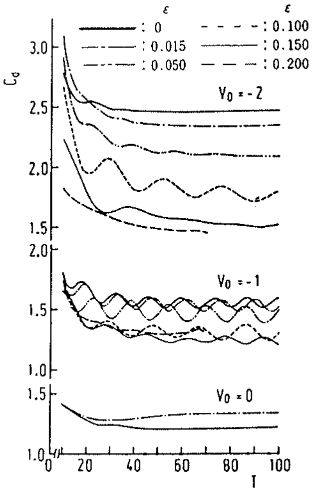

Fig.2 Variation of drag coefficient with time

to be in stationary state as mentioned in the former report.[2] When the state shifts suddenly to an unstable one, $\bar{C}_l$ and $\bar{C}_d$ are determined by referring to those in the stable state. In the figures, the curves of ($V_0 = -1$, $\varepsilon = 0.2$) and ($V_0 = -2$, $\varepsilon = 0.2$) are shown only up to $T = 70$ because C_l and C_d diverge after T= 60. When V_0=0, both $\bar{C}_l$ and $\tilde{C}_l$ increase with ε, and $\tilde{C}_l$ decreases as ε increases after it reaches a maximum for $\varepsilon > 0.015$ according to Tamura et al.[4] Here $\tilde{C}_l$ is

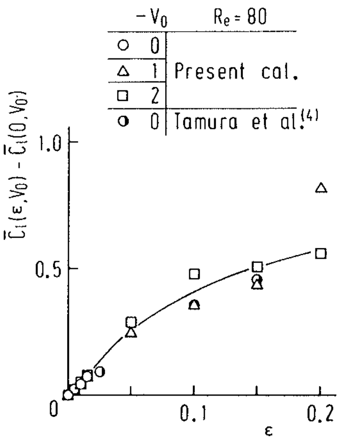

Fig.3 Relation of increment of $\bar{C}_l$ to ε

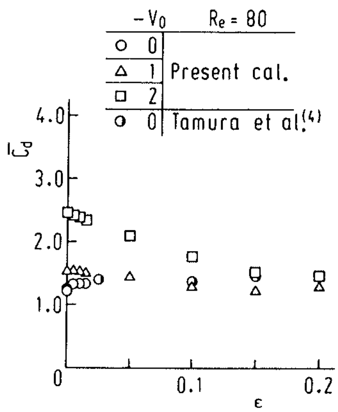

Fig.4 Relation of $\bar{C}_d$ to ε

the amplitude of periodic fluctuation of C_l. When $V_0 = -1$, both $\bar{C}_l$ and $\tilde{C}_l$ increase with ε, and $\tilde{C}_l$ reaches a maximum at $\varepsilon = 0.05$ and decreases as ε increases for $\varepsilon > 0.05$. When $V_0 = -2$, $\bar{C}_l$ increases with ε, and $\tilde{C}_l$ begins to increase from about $\varepsilon = 0.05$, reaches a maximum, and decreases as ε increases. In the case of uniform flow ($\varepsilon = 0$), no periodic fluctuation of C_l is observed when $V_0 = -2$. In the case of uniform shear flow, the period of fluctuation of C_l changes when the value of ε changes at any value of V_0. $\tilde{C}_d$, the amplitude of C_d, also changes its value and period as V_0 and ε change, and shows the same tendency as $\tilde{C}_l$ does (Fig.2). Value of $\tilde{C}_d$ is smaller than that of $\tilde{C}_l$ as in a uniform flow. As regards $\bar{C}_d$, $\bar{C}_d$ increases with ε when $V_0 = 0$, while $\bar{C}_d$ decreases as ε increases when $V_0 \neq 0$. This decrease of $\bar{C}_d$ is larger when $|V_0|$ is larger. Consequently, as ε increases, $\bar{C}_d$ approaches asymptotically a value of about $1.3 \sim 1.5$ at any value of V_0 if $0 \leqq |V_0| \leqq 2$. $\bar{C}_l$ and $\bar{C}_d$ are shown against ε in Figs.3 and 4, respectively. The ordinate of Fig.3, however, is $\bar{C}_l(\varepsilon, V_0)$ at $\varepsilon \neq 0$ minus $\bar{C}_l(0, V_0)$ at $\varepsilon = 0$, keeping V_0 constant. Figure 3 shows that the increment of $\bar{C}_l$ due to ε seems to be independent of V_0. The value at $V_0 = -1$ and $\varepsilon = 0.2$ in the figure is considered less reliable in accuracy since C_l at those V_0 and ε diverges suddenly for $T > 60$. Figure 4 confirms that $\bar{C}_d$ approaches

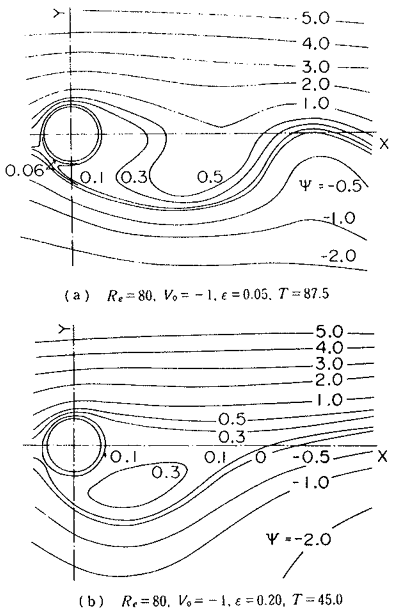

Fig.5 Stream lines around the rotating cylinder in uniform shear flow

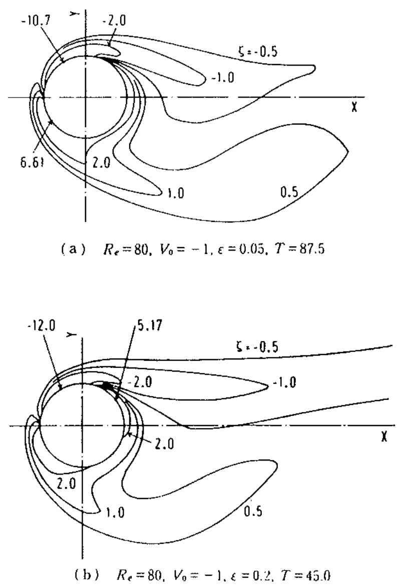

Fig.6 Equivorticity lines around the rotating cylinder in uniform shear flow

asymptotically a value of about 1.3∿1.5 as ε increases irrespective of V_0. The proportion of the pressure components in $\bar{C}_l$ and $\bar{C}_d$ is nearly independent of ε and large as in the case of $\varepsilon = 0$ (not shown here). Figures 5(a), (b) and 6(a), (b) show the stream line distributions and equi-vorticity lines, respectively. The figure (a) corresponds to the case that the amplitude of the fluctuation is largest ($\varepsilon = 0.05$) when $V_0 = -1$ while (b) corresponds to the case that $\tilde{C}_l$ is virtually zero ($\varepsilon = 0.2$) when $V_0 = -1$. Figure 5 shows that the flow field is almost steady, and that the wake deflects upwards at $\varepsilon = 0.2$. The latter is by the same reason as mentioned in the former report,[2] that is, the rotation of the cylinder ($V_0 < 0$) supplies the wake behind the cylinder with positive vorticity while the velocity gradient ($\varepsilon > 0$) supplies the wake with negative vorticity. This is also understood by comparing Fig.6(a) ($\varepsilon = 0.05$) with (b) ($\varepsilon = 0.2$). Figure 6(a) is the case that the part of positive ζ is in average a little wider than that of negative ζ though the areas of the positive and negative parts fluctuate periodically with time. On the other hand, (b) is the case that the negative part of ζ is steadily wider. Consequently, the wake deflects downwards in Fig.5(a), while it deflects upwards in (b). Therefore, $\bar{C}_d$ decreases with a decrease of the induced drag on the cylinder as ε increases when the cylinder is rotating (Fig.4). The position of the stagnation point is hardly affected by ε, and determined by V_0 as in a uniform flow.

Figure 7 shows the amplitude of fluctuation $\tilde{C}_l$ at respective V_0 against ε. It is seen from the figure that there exists a certain ε at which $\tilde{C}_l$ is the maximum for V_0, and that such ε is larger when $|V_0|$ is larger. The maximum value is smaller when $|V_0|$ is larger. This implies that there exists an unbalanced point between strengths of positive and negative vortex sheets (or

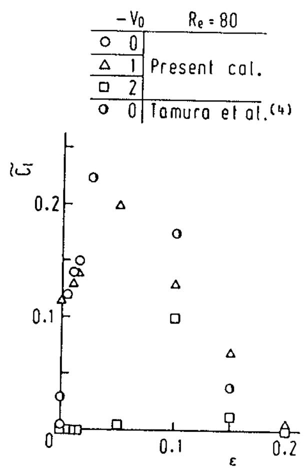

Fig.7 Relation of amplitude $\tilde{C}_l$ to ε

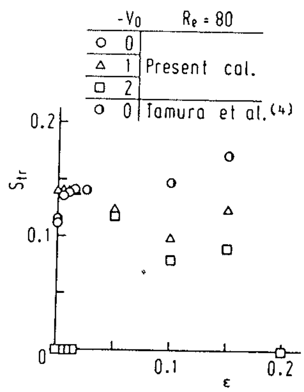

Fig.8 Relation of the Strouhal number S_{tr} to ε

streets) in the wake to make $\tilde{C}_1$ maximum, though the intervals of V_0 and ε used for the present computation are too broad to discuss this problem further. Figure 8 shows the Strouhal number S_{tr} of $\tilde{C}_1$. The present result at $V_0 = 0$ is connected smoothly with that by Tamura et al.[4] When $V_0 \fallingdotseq 0$, S_{tr} has a minimum ($\fallingdotseq 0$), at which the value of ε seems to become larger when $|V_0|$ is larger.

6. Conclusions

Numerical solutions for $R_e = 80$ were obtained on the flows around a circular cylinder with V_0 of $0 \sim -2$ in a uniform shear flow of ε of $0 \sim 0.2$. The following are conclusions.

(1) $\bar{C}_1$ increases with ε(>0) when V_0 (<0) is constant. The increment of $\bar{C}_1$ due to ε is independent of V_0. $\bar{C}_d$ approaches a value of about $1.3 \sim 1.5$ as ε increases for any V_0 for $-2 \leqq V_0 \leqq 0$.

(2) When V_0 is kept constant, the wake deflects gradually from downwards to upwards as ε increases. This is due to an induced velocity by the vorticity in the wake. The position of the stagnation point, however, is nearly independent of ε.

(3) $\tilde{C}_1$ has a maximum, for a certain constant V_0, at a certain value of ε. The value of this ε is larger when $|V_0|$ is larger. The maximum value of $\tilde{C}_1$ is smaller when $|V_0|$ is larger. $\tilde{C}_d$ shows the same tendency as $\tilde{C}_1$ but the tendency is milder than that of $\tilde{C}_1$.

(4) The flow field is steady for a certain combination of V_0 and ε even at such a Reynolds number of 80 that the vortex streets are formed behind a still cylinder. The Strouhal number S_{tr} of $\tilde{C}_1$ has a minimum ($\fallingdotseq 0$) when $V_0 \fallingdotseq 0$, at which the value of ε seems to be larger when $|V_0|$ is larger.

References

(1) Ozaki, M., et al., Preprint of Japan Soc. Mech. Engrs. (in Japanese), No.855-1 (1985-3), 118.

(2) Yoshino, F., et al., Rep. of Faculty of Engng. Tottori Univ., Vol.15, No.1 (1984-10), 1.

(3) Frenkiel, F.N. and Temple, G.(ed.), Applid Mathematics and Mechanics, Vol.8 (1964), 12, Academic press.

(4) Tamura, H., et al., Trans. Japan Soc. Mech. Engrs. (in Japanese), Vol.46, No.404 (1980-4), 555.