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Corrections to "The chromatic E_1-term $H^1M_2^1$ and its application to the homotopy of the Toda-Smith spectrum $V(1)$"

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§ 1. Introduction and statement of results

In the proof of Theorem A of [1, p.81], we claimed that all the elements in Proposition 4.14 are linearly independent. Chun-Nip Lee, however, points out that this is incorrect. This means that there are some more differentials to be computed of the Bockstein spectral sequence. The completion of the computation requires replacement of some elements in [1, (4.10), (4.11)] with suitable ones so that the proof of Theorem A holds true throughout all our discussion. Because of this correction, we change Theorem A and C of [1] to the following ones. Stating them needs some notations, which are the same as those in [1] except the definition of the integers $\lambda(k)$ and $\mu(k)$. Recall that the coefficient ring of the Brown-Peterson spectrum at a prime p is a polynomial algebra $Z_{(p)}[v_1, v_2, \cdots]$ over the generators v_i with degree $|v_i| = 2p^i - 2$. For the generator v_n with $n \geq 1$, consider the polynomial algebras

(1.1) $$k(n)_* = Z/p[v_n] \quad \text{and} \quad K(n)_* = Z/p[v_n, v_n^{-1}]$$

and $k(2)_*$-modules

(1.2)

$L_n(x)$ — *the cyclic $k(2)_*$-module generated by the element x with $v_2^n x = 0$, and*

$L\{x_j\}$ — *the $k(2)_*$-module isomorphic to*

$$K(2)_*/k(2)_*$$

with Z/p-basis $\{x_j\}$ such that the $k(2)_*$-action is given by $v_2 x_j = x_{j-1}$ and $x_j = 0$ for $j \leq 0$.

We define integers $\nu(k)$ and $\varepsilon(k)$ for an integer k by

$$\nu(k) = \max\{n : p^n | k\} \quad \text{and} \quad \varepsilon(k) = \frac{1}{2}(1 - (-1)^k),$$

and integers $a(i)$, $b(i)$, $c(i)$ and $e(i)$ for a non-negative integer i by

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$$a(i) = p^i + (p^{i-1} - 1)/(p+1) \quad \text{for odd } i \geq 1, \text{ and}$$
$$= pa(i-1) \quad \text{for even } i \geq 2;$$
(1.3) $$b(i) = p^{i-2}(p^2 + p + 1) \quad \text{for } i \geq 2;$$
$$c(i) = p^{i-2}(p^2 - p - 1) \quad \text{for } i \geq 2; \text{ and}$$
$$e(i) = (p^i - 1)/(p - 1).$$

Using these intgers we give further integers:

$$\lambda(k) = 2 \quad \text{for } k \text{ with } p \nmid k(k-1),$$
$$= 2p \quad \text{for } k = up + 1 \text{ with } p \nmid u(u-1),$$
$$= p^2 + 1 \quad \text{for } k = up^2 + 1 \text{ with } p \nmid u,$$
$$= a(l) + p \quad \text{for } k = up^l + 1 \text{ with } l \geq 3 \text{ and } p \nmid u,$$
(1.4) $$= a(l) + 1 \quad \text{for } k = up^l + e(l) \text{ with } l \text{ even } \geq 2 \text{ and } p \nmid u - 1,$$
$$= a(l) + 2 \quad \text{for } k = up^l + e(l) \text{ with } l \text{ odd } \geq 3 \text{ and } p \nmid u - 1,$$
$$= p + 1 \quad \text{for } k = up \text{ with } p \nmid u,$$
$$= b(l) - 1 \quad \text{for } k = up^l \text{ with } l \text{ even } \geq 2 \text{ and } p \nmid u,$$
$$= b(l) - p + 1 \quad \text{for } k = up^l \text{ with } l \text{ odd } \geq 3 \text{ and } p \nmid u;$$
$$\mu(k) = 2 \quad \text{for } k \text{ with } p \nmid k(k+1),$$
$$= 2p \quad \text{for } k = up \text{ with } p \nmid u(u+1),$$
$$= 2p^2 - p + 1 \quad \text{for } k = up^2 \text{ with } p \nmid u(u+1),$$
(1.5) $$= 2a(l) - p + 2 \quad \text{for } k = up^l \text{ with } l \text{ odd } \geq 3 \text{ and } p \nmid u(u+1),$$
$$= 2a(l) \quad \text{for } k = up^l \text{ with } l \text{ even } \geq 4 \text{ and } p \nmid u(u+1),$$
$$= (p-1)a(r+1) \quad \text{for } (up^2 - 1)p^r \text{ with } r \geq 0; \text{ and}$$
(1.6) $$\tilde{a}(k) = a(l) \quad \text{for } k = up^l \text{ with } p \nmid u.$$

These integers enable us to state Theorem A to be the same as that of [1].

THEOREM A. *Let p be a prime ≥ 5. Then the E_1-term $H^1 M_2^1$ of the chromatic spectral sequence is the direct sum of $k(2)_*$-modules:*

(a) $L\{z_j\}$, $L\{x_{\varepsilon,j}\}$ (for $\varepsilon = 0, 1$), *and* $L\{\tilde{z}_j\}$;
(b) $L_{p-1}\langle\chi(k)\rangle$ *for* $k \in \boldsymbol{Z}$;
(c) $L_{\lambda(k)}\langle\varphi(k)\rangle$ *for* $k \in \boldsymbol{Z}$;
(d) $L_{\mu(k)}\langle\psi(k)\rangle$ *for* $k \in \boldsymbol{Z}$; *and*
(e) $L_{\tilde{a}(k)}\langle\zeta(k)\rangle$ *for* $k \in \boldsymbol{Z}$.

Here degrees of these generators are given by:

$$|z_j| = -2j(p^2-1),$$

$$|x_{\varepsilon,j}| = 2p^\varepsilon(p-1) - 2j(p^2-1) \qquad \text{for } \varepsilon = 0, 1,$$

$$|\tilde{z}_j| = 2(p+1-j)(p^2-1),$$

$$|\chi(k)| = 2(kp-1)(p^3-1) + 2p^2(p-1) - 2(p-1)(p^2-1),$$

$$|\varphi(k)| = 2k(p^3-1) + 2p^{\varepsilon(\nu(k)+1)}(p-1) - 2\lambda(k)(p^2-1),$$

$$|\psi(k)| = 2k(p^3-1) + 2p^{\varepsilon(\nu(k))}(p-1) - 2\mu(k)(p^2-1), \text{ and}$$

$$|\zeta(k)| = 2k(p^3-1) - 2\tilde{a}(k)(p^2-1).$$

We rewrite Theorem C in [1] by the following

THEOREM C. *Let p be a prime ≥ 7, and r and s non-negative integers with $p \nmid s$. Then in $\pi_* V(1)$, we have*

(a) $\gamma'_{[sp^r]}\beta_1 \neq 0 \neq \gamma'_{[sp^r/2]}\beta_1$ *if r is even or $p \nmid s+1$,*

(b) $\gamma'_{[sp^r]}\beta_2 \neq 0 \neq \gamma'_{[sp^r/2]}\beta_3$ *if $r = 1$; if $p \nmid (s+1)(s+2)$, $p^2 | s+1$, $p^2 | s+2$ or $p^2 | s+2+p$ for odd $r \geq 3$; or if $p^2 \nmid s+1+p$ or $p^3 | s+1+p$ for even $r \geq 0$; and*

(c) $\gamma'_{[sp^r/2]}\beta_2 \neq 0$ *if r is even, $r = 1$, $p \nmid s+1$ or $p^2 | s+1$.*

Here the integer r in $\gamma'_{[sp^r/2]}$ is positive.

In the next section we prepare some elements and a lemma in order to replace Proposition 4.14 in [1] by a suitable proposition, and then we show the substituted proposition.

There seems no error in §§2–3 of [1]. On the correction of [1, §4] we state in the next section and we correct some statements in §5 of [1] as follows: We replace the if-clause of the statement on the 8th line in [1, p. 82]: "$\lambda GB_2 \neq 0$ if $r \neq 0, 2$ or $p^2 \nmid s + p + 1$" by

(1.7) if $r = 1$; if $p \nmid (s+1)(s+2)$, $p^2 | s+1$, $p^2 | s+2$, or $p^2 | s+2+p$ for odd $r \geq 3$; or if $p^2 \nmid s+1+p$ or $p^3 | s+1+p$ for even $r \geq 0$.

This replacement is accordingly applied to the if-clause of the second inequality of [1, Th. 5.2].

We also replace the if-clause of the second inequation of [1, Cor. 5.4] by (1.7) with $r \geq 1$. Add to the end of [1, Th. 5.5] the sentence "if r is even, $r = 1$, $p \nmid s+1$ or $p^2 | s+1$", and we complete the correction.

The author would like to thank Professor Chun-Nip Lee for not only pointing out the error but also suggesting him how the answer should be.

§2. More differentials

The computation in [1, §4] does not complete the determination of the differentials of the Bockstein spectral sequence associated to the short exact sequence $0 \to M_3^0 \to M_2^1 \xrightarrow{v_2} M_2^1 \to 0$. We give some more differentials to complete the computation in this section.

Recall [1, (3.9), (3.18)] the element P_{21} and ω_3 and define

$$\bar{P}_{21} = v_2^{p^2} P_{21} - \omega_3 \quad \text{and} \quad \bar{P}'_{21} = \bar{P}_{21} + v_2^{p^2} z t_1^p. \tag{2.1}$$

Then the following lemma is a corollary of Lemmas 3.10 and 3.20 of [1].

LEMMA 2.2. *$d_1 \bar{P}_{21} \equiv v_2^{p^2} z \otimes t_1^p - v_2^{p^2+p} k_0 - v_2^{p^2+p+1} b_0$ mod I_2, and $d_1 \bar{P}'_{21} \equiv - v_2^{p^2} t_1^p \otimes z - v_2^{p^2+p} k_0 - v_2^{p^2+p+1} b_0$ mod I_2.*

Now redefine some of the elements $\varphi(m/j)$ and $\psi(m/j)$ in [1, (4.10), (4.11)]:

$$\begin{aligned} &\varphi(up^l + 1/j) = X_l^u z/v_2^j + u X_{l-1}^{up-1} \bar{P}'_{21}/v_2^{j+p^2-a(l)} && \text{for } l \text{ even} \geq 4, \\ &\varphi(tp^r/j) = \sigma_t X_r/v_2^j + t v_3^{c(t,r)} \bar{P}_{21}/v_2^{j-b(r)+p^2+p+1} && \text{for } r \text{ even} \geq 6, \text{ and} \\ &\psi(sp^r/j) = \tau_s X_{r+1}/v_2^j + 2s(s+1) v_3^{sp^r-2p^{r-1}} \bar{P}'_{21}/v_2^{j-2a(r)+p^2+p} && \text{for } r \text{ even} \geq 4. \end{aligned} \tag{2.3}$$

Then we have the following

PROPOSITION 2.4. *Let $\delta: H^1 M_2^1 \to H^2 M_3^0$ be the connecting homomorphism associated to the exact sequence $0 \to M_3^0 \to M_2^1 \xrightarrow{v_2} M_2^1 \to 0$. Then the δ-image of the elements of (2.3) are:*

(a) $\delta\varphi(up^l + 1/a(l) + p) = -u v_3^{up^l - p^{l-1}} k_0$ *for l even ≥ 4.*

(b) $\delta\varphi(tp^r/b(r) - 1) = -\dfrac{3t}{2} v_3^{c(t,r)} k_0$ *for r even ≥ 6.*

(c) $\delta\psi(sp^r/2a(r)) = -s(s+1) v_3^{sp^r - 2p^{r-1}} k_0$ *for r even ≥ 4.*

PROOF. The definition of δ shows that $\delta x/v_2^j = y$ if $d_1 x \equiv v_2^j y$ mod $J(v_2^{j+1})$. Since $d_1 z \equiv 0$ mod I_2, we have $d_1 X_l^u z \equiv u v_2^{a(l)} v_3^{up^l - p^{l-1}} t_1^p \otimes z$ mod $J(a(l)+p+1)$ by [1, Prop. 2.18]. Lemma 2.2 shows $u d_1 v_2^{a(l)-p^2} X_{l-1}^{up-1} \bar{P}'_{21} \equiv u v_2^{a(l)-p^2} v_3^{up^l - p^{l-1}}(- v_2^{p^2} t_1^p \otimes z - v_2^{p^2+p} k_0)$, and we have (a). We compute $d_1 \sigma_t X_r \equiv - t v_2^{b(r)-p-1} v_3^{c(t,r)}(z \otimes t_1^p + v_2^p k_0)$ mod $J(b(r))$ by noticing the equation $b(r) = a(r) + a(r-1) + 1$, using [1, Prop. 2.18 and Lemma 3.17], and $d_1 \tau_s X_{r+1} \equiv 2s(s+1) v_2^{2a(r)-p} v_3^{sp^r - 2p^{r-1}} t_1^p \otimes z +$

$\binom{s+1}{2} v_2^{2a(r)} v_3^{sp^r - sp^{r-1}} k_0'$. Again use Lemma 2.2, and we obtain (b) and (c). q.e.d.

Now we correct §4 of [1]. The fourth equation of [1, (4.10)] is replaced by $\varphi(up^2 + 1/j) = X_2^u z/v_2^j + uX_1^{up-1} P_{21}'/v_2^{j-a(l)}$ and the first one of (2.3), and the last one of [1, (4.10)] is only for odd ≥ 3 and add the second one of (2.3) for even $r \geq 6$. The third one of [1, (4.11)] is only for $r = 2$ or odd $r \geq 3$ and we add the last equation of (2.3) to [1, (4.11)] for even $r \geq 4$. The fifth equation in 2 of [1, Prop. 4.14] is only for $l = 2$, and for even $l \geq 4$ we use (a) of Proposition 2.4. Replace the last one in 2 of [1, Prop. 4.14] by (b) of Proposition 2.4. We add Proposition 2.4 (c) between the third and the fourth equations in 3 of [1, Prop. 4.14] and replace the third one by $\delta\psi(sp^2/2p^2 - p + 1) = \binom{s+1}{2} v_3^{sp^2 - 2p} g_1$.

References

[1] K. Shimomura, The chromatic E_1-term $H^1 M_2^1$ and its application to the homotopy of the Toda-Smith spectrum $V(1)$, J. Fac. Educ. Tottori Univ. (Nat. Sci.), **40** (1991), 63–83.