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On the E_2-term of the Novikov spectral sequence for a Thom spectrum

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§1. Introduction

In his paper [2], Mahowald shows the way to construct a ring spectrum from Thom complexes of a fibration. As an example, there is a ring spectrum $X\langle 1\rangle$ obtained from the fibration classified by the canonical generator $\Omega S^2 \to BO$. Consider the Brown-Peterson homology $BP_*(-)$ at the prime 2, whose coeffcient is the polynomial algebra $BP_*(S)=Z_{(2)}[v_1, v_2, \cdots]$, and we see that

$$BP_*(X\langle 1\rangle)=BP_*/(2)[t_1]$$

as a subcomodule algebra of $BP_*BP/(2)$, where $BP_*BP=BP_*[t_1, t_2, \cdots]$. We also consider the ring spectrum $E(2)$ whose coefficient is a polynomial ring $Z_{(2)}[v_1, v_2, v_2^{-1}]$. In this note we study about the E_2-term of the Adams-Novikov spectral sequence for the $E(2)$-localization $L_2X\langle 1\rangle$ of the spectrum $X\langle 1\rangle$ and obtain the first few lines. Here the E_2-term is $\mathrm{Ext}^s_{BP_*BP}(BP_*, (v_2^{-1}BP_*/(2))[t_1])$. In order to state our theorem we prepare some notation. Let

$$\delta: \mathrm{Ext}^s_{BP_*BP}(BP_*, (v_2^{-1}BP_*/(2, v_1^\infty))[t_1]) \to \mathrm{EXt}^s_{BP_*BP}(BP_*, (v_2^{-1}BP_*/(2))[t_1])$$

denote the boundary homomorphism associated to the exact sequence

$$0 \longrightarrow (v_2^{-1}BP_*/(2))[t_1] \longrightarrow (v_1^{-1}v_2^{-2}BP_*/(2))[t_1] \longrightarrow (v_2^{-1}BP_*/(2, v_1^\infty))[t_1] \longrightarrow 0.$$

We also use the integers A_k and c_k defined by

$$A_0=1, \quad A_{2k}=4A_{2k-2}+2, \quad \text{and} \quad A_{2k+1}=2A_{2k}; \text{and}$$

$$c_0=-1, \quad \text{and} \quad c_{k+1}=4c_k+4,$$

and an algebra $K(2)_*=\boldsymbol{Z}/2[v_2, v_2^{-1}]$.

THEOREM. *The E_2-term of the Adams-Novikov spectral sequence for $L_2X\langle 1\rangle$ at the filtration degree less than 3 is given as follows:*

0) $E_2^{0,*}=\boldsymbol{Z}/2[v_1, v_2]$

1) $E_2^{1,*}$ *is a direct sum of* $\boldsymbol{Z}/2[v_1, v_2]\{h_{20}\}$ *and* M.

2) $E_2^{2,*}$ *is a direct sum of* $M\{h_{31}\}$, *a free* $K(2)_*$*-module generated by*

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$$\delta v_3^t h_{30}/v_1 \quad \text{and} \quad \delta v_3^t \rho/v_1$$

for $t \geq 0$, *and cyclic* $K(2)_*[v_1]$*-modules generated by*

$$\delta v_3^{2s+1} h_{20}/v_1,\ \delta v_3^{2t} h_{20}/v_1^2 \quad (t \neq 0) \quad \text{and} \quad \delta v_2^t h_{21}/v_1$$

for $s \geq 0$ *and* $t \notin G = \{2^{2k+1}s + c_k + 1 | k \geq 0\}$.

Here h_{ij} *and* ρ *are the elements represented by cycles* $t_i^{2^j}$ *and* $v_2^5 t_4 + t_4^2$ *of the cobar complex, and* M *denotes a direct sum of cyclic* $K(2)_*[v_1]$*-modules generated by*

$$\delta x_n^t / v_1^{A_n}$$

for $n \geq 0$, *an odd integer* $t > 0$, *and some element* x_n *such that* $x_n \equiv v_3^{2^n} \bmod (2, v_1)$*(see §3).*

This proved by using v_1 Bockstein spectral sequence. First we give the structure of the E_2-term of the Adams-Novikov spectral sequence for the $E(2)$-localization $L_2N(1, 1)$ of the cofiber $N(1, 1)$ of the map $v_1: \Sigma^2 X\langle 1\rangle \to X\langle 1\rangle$, which is a tensor product of an exterior and a polynomial algebras (see Prop. 2.5). We note that we study in [3] about the homotopy group of $L_2N(1, 1)$ and see that the E_∞-term of the spectral sequence is a tensor product of an exterior and a stunted polynomial. The E_2-terms $E_2^{s,*}$ for any s are non-trivial in our case, though most of them are trivial at an odd prime p. In a same way as the proof of our theorem it seems to be computed the E_2-term for higher filtration, but here we only give some differentials for them (Lemma 3.7).

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§2. Change of rings

In this section we give some theorems for the comodule $(v_2^{-1}BP_*/(2, v_1))[t_1]$ similarly to those of [6] and [7].

Let BP denote the Brown-Peterson spectrum at the prime 2 and $K(k)$ the Morava K-spectrum whose coefficient rings are $\boldsymbol{Z}_{(2)}[v_1, v_2 \cdots]$ and $\boldsymbol{Z}/(2)[v_k, v_k^{-1}]$, respectively, on Hazewinkel's generators v_k's with degree $2^{k+1}-2$. Then $\Gamma = BP_*BP = BP_*[t_1, t_2, \cdots]$ for the generators t_i's with degree $2^{i+1}-2$, and we define $\Sigma(k) = K(k)_*K(k)$ to be $K(k)_* \otimes_{BP_*} BP_*BP \otimes_{BP_*} K(k)_*$. These give rise to Hopf algebroids (BP_*, Γ) and $(K(k)_*, \Sigma(k))$ and the natural map $\Gamma \to \Sigma(k)$ of Hopf algebroids. For a Hopf algebroid (A, L), the Ext-group $\mathrm{Ext}_L^s(A, M)$ of a comodule M with structure map ψ is defined to be a cohomology of the cobar complex $\Omega_L^* M$ with $\Omega_L^s M = M \otimes_A L \otimes_A \cdots \otimes_A L$ (s copies of L) provided by the differential $d: \Omega_L^s M \to \Omega_L^{s+1} M$ given by $d(m \otimes x) = \psi m \otimes x + \sum_{k=1}^s (-1)^k m \otimes \Delta_k x - (-1)^s m \otimes x \otimes 1$ where $\Delta_k = 1_{k-1} \otimes \Delta \otimes 1_{s-k}$ for the identity map $1_l: L^{\otimes l} \to L^{\otimes l}$. We have the following change of rings theorem:

(2.1) [4, Th. 2.10] *If M is v_1-local and $2M = 0$, then*

$$\mathrm{Ext}^*_{\Gamma}(BP_*, M) \cong \mathrm{Ext}^*_{\Sigma(1)}(K(1)_*, K(1)_* \otimes_{BP_*} M)$$

under the natural map, and if M is v_2-local and $2M = 0 = v_1 M$, then

$$\mathrm{Ext}^*_{\Gamma}(BP_*, M) \cong \mathrm{Ext}^*_{\Sigma(2)}(K(2)_*, K(2)_* \otimes_{BP_*} M)$$

under the natural map.

Then for Hopf algebroid $(K(1), \Sigma(1))$, we have

(2.2) [8, Cor. 6.5.6] $\mathrm{Ext}_{\Sigma(1)}(K(1)_*, K(1)_*[t_1]) = K(1)_*[v_2] \otimes E(h_{20})$.

Consider the Hopf algebroid $(K(2)_*, \Sigma) = (K(2)_*, \Sigma(2)/(t_1))$ such that the projection $\Sigma(2) \to \Sigma$ is a map of Hopf algebroids. We then have the following

(2.3) [8, Th. 6.5.5]

$$\mathrm{Ext}^*_{\Sigma(2)}(K(2)_*, K(2)_*[t_1]) = K(2)_*[v_3] \otimes_{K(2)_*} \mathrm{Ext}_{\Sigma}(K(2)_*, K(2)_*).$$

As in [8, Th. 6.3.7], the manner developed in [6] and [7] to compute $\mathrm{Ext}_{\Sigma(2)}(K(2)_*, K(2)_*)$ is applied to obtain

(2.4) *There is a spectral sequence*

$$E(h_{20}, h_{21}, h_{30}, h_{31}, h_{40}, h_{41}) \otimes P(b_{20}, b_{21}) \Longrightarrow \mathrm{Ext}_{\Sigma}(K(2)_*, K(2)_*)$$

with differential

$$d_2 h_{4j} = h^2_{2,j} + b_{2,j+1},$$

where h_{ij} and b_{2j} for $j \in \mathbf{Z}/2$ are represented by $t_i^{2^j}$ and $t_2^{2^j} \otimes t_2^{2^j}$, respectively, and $h^2_{2j} = b_{2j}$, *in the cobar complex $\Omega^*_{\Sigma(2)} K(2)_*$. Here E and P stand for the exterior and the polynomial algebras over $K(2)_*[v_3]$.*

This shows the following

PROPOSITION **2.5.**

$$\mathrm{Ext}_{BP_*BP}(BP_*, (v_2^{-1} BP_*/(2, v_1))[t_1]) = K(2)_*[v_3, h_{20}] \otimes \Lambda(h_{21}, h_{30}, h_{31}, \rho).$$

Here h_{2i}, h_{3i} and ρ is represented by $t_2^{2^i}$, $t_3^{2^i}$ and $v_2^5 t_4 + t_4^2$, respectively.

§3. The Bockstein spectral sequence

In this section we compute the Bockstein spectral sequence to give the chromatic E_1-terms for computing our target $\mathrm{Ext}_{BP_*BP}(BP_*, (v_2^{-1} BP_*/(2))[t_1])$, and consider the following $\Gamma = BP_*BP$-comodules:

$$A = v_2^{-1} BP_*/(2), \; B = v_1^{-1} A, \; C = B/A, \quad \text{and} \quad D = v_2^{-1} BP_*/(2, v_1),$$

and the notation

$$H^{s,t}M = \mathrm{Ext}_\Gamma^{s,t}(BP_*, M).$$

These notations give rise to short exact sequences:

$$0 \longrightarrow A \xrightarrow{\subset} B \longrightarrow C \longrightarrow 0, \text{ and}$$

$$0 \longrightarrow D \longrightarrow C \xrightarrow{v_1} C \longrightarrow 0,$$

which induces the long exact sequences:

$$\begin{aligned}(3.1)\qquad &0 \longrightarrow H^0A \longrightarrow H^0B \longrightarrow H^0C \xrightarrow{\delta} H^1A \longrightarrow \cdots \\ &\longrightarrow H^kA \longrightarrow H^kB \longrightarrow H^kC \xrightarrow{\delta} H^{k+1}A \longrightarrow \cdots\end{aligned}$$

and

$$\begin{aligned}(3.2)\qquad &0 \longrightarrow H^0D \longrightarrow H^0C \longrightarrow H^0C \xrightarrow{\delta} H^1D \longrightarrow \cdots \\ &\longrightarrow H^kA \longrightarrow H^kC \longrightarrow H^kC \xrightarrow{\delta} H^{k+1}D \longrightarrow \cdots\end{aligned}$$

We call the sequences (3.1) and (3.2) the chromatic and the Bockstein exact sequences, respectively.

Now define integers:

$$\begin{aligned} a_0 &= 1, \qquad a_{k+1} = 4a_k + 2; \\ b_0 &= 0, \qquad b_{k+1} = 4b_k + 3; \\ c_0 &= -1, \quad c_{k+1} = 4c_k + 4; \end{aligned}$$

and element of $v_2^{-1}BP_*$:

$$\begin{aligned} x_0 &= v_3 \\ x_1 &= v_3^2 - v_1^2 v_2^{-1} v_4 \\ x_2 &= x_1^2 - v_1^3 v_2^6 v_3 - v_1^4 v_2^{-6} v_3^4 x_1 - v_1^6 v_2^{-3} v_5 + v_1^6 v_2^{-5} v_3 v_4^2 \end{aligned}$$

and

$$x_{2k+1} = x_{2k}^2 - v_1^{2a_k} v_2^{-1+2b_k} v_3^{2+2c_k} v_4$$

$$\begin{aligned} x_{2k+2} = x_{2k+1}^2 - v_1^{4a_k} v_2^{4b_k} v_3^{4c_k} (&v_1^{-6} v_2^{10} \overline{x_2} \\ &+ v_1^{-1} v_2^6 v_3^5 + v_1^2 v_2^{-3} v_3^4 v_5 + v_1^2 v_2^{-5} v_3^5 v_4^2 \\ &+ v_1^{-10} v_2^2 x_3 + v_1^2 v_2^7 v_3^2 v_4 + v_2^{-6} v_3^8 x_1) \end{aligned}$$

for $k \geq 1$, where $\overline{x_2} = x_1^2 - v_1^3 v_2^6 v_3 - v_1^5 v_2^3 x_1$. The 0-th differential d of the cobar complex is $\eta_R - \eta_L$ where η_L is the inclusion $BP_* \to BP_*BP$, since comodules which we treat here have the structure maps induced from the right unit map η_R. Then we have

LEMMA 3.3. *In* $(v_2^{-1}BP_*/(2))[t_2, t_3, \cdots]$,

$$dx_0 \equiv v_1 t_2^2 - v_1^4 t_2$$
$$dx_1 \equiv v_1^2 v_2^3 t_2 + v_1^3 v_2^{-1} t_3^2 \quad \mathrm{mod}\ (v_1^4)$$
$$dx_{2k} \equiv v_1^{a_k} v_2^{b_k} v_3^{1+c_k} t_2^2 + v_1^{a_k} v_2^{2+b_k} v_3^{c_k} t_3 \quad \mathrm{mod}\ (v_1^{1+a_k})$$
$$dx_{2k+1} \equiv v_1^{2a_k} v_2^{3+2b_k} v_3^{2+2c_k} t_2 + v_1^{2a_k} v_2^{4+2b_k} v_3^{2c_k} t_3^2 + v_1^{1+2a_k} v_2^{-1+2b_k} v_3^{2+2c_k} t_3^2 \quad \mathrm{mod}\ (v_1^{2+2a_k})$$

PROOF. This follows from a direct calculation using

$$\begin{aligned} dv_1 &= 0 \\ dv_2 &= 0 \\ dv_3 &= v_1 t_2^2 - v_1^4 t_2 \\ dv_4 &\equiv v_2 t_2^4 - v_2^4 t_2 + v_1 t_3^2 - v_1^2 v_3 t_2^2 \quad \mathrm{mod}\ (v_1^5) \\ dv_5 &\equiv v_3 t_2^8 - v_3^4 t_2 + v_2 t_3^4 - v_2^8 t_3 \quad \mathrm{mod}\ (v_1), \end{aligned} \tag{3.4}$$

which are verified easily by Hazewinkel's and Quillen's formulae (*cf.* [8]): $v_n = pm_n - \sum_{i=1}^{n-1} m_i t_{n-i}^{2^i}$ and $\eta_R m_n = \sum_{i+j=n} m_i t_j^{2^i}$. The first one in the lemma is the immediate consequence of the third equality in (3.4). Noting that $dx \equiv y$ mod (a) implies $dx^2 \equiv y^2$ mod (a^2) enables us to go forward by inductive steps. The first result shows $dv_3^2 \equiv v_1^2 t_2^4$ mod (v_1^5) and a direct calculation brings $dv_1^2 v_2^{-1} v_4 \equiv v_1^2 v_2^{-1}(v_2 t_2^4 - v_2^4 t_2 + v_1 t_3^2 - v_1^2 v_3 t_2^2)$ mod (v_1^5), the sum of which is the second one of the lemma.

This shows $dx_1^2 \equiv v_1^4 v_2^6 t_2^2 + v_1^6 v_2^{-2} t_3^4 + v_1^8 v_2^{-2} v_3^2 t_2^4$ mod (v_1^{10}). Then we get

$$d\overline{x_2} \equiv v_1^6 v_2^{-2} t_3^4 + v_1^8 v_2^2 t_3^2 + v_1^8 v_2^{-2} v_3^2 t_2^4 \ \mathrm{mod}\ (v_1^9)$$

by the results $dv_1^3 v_2^6 v_3 \equiv v_1^3 v_2^6(v_1 t_2^2 - v_1^4 t_2)$ and $dv_1^5 v_2^3 x_1 \equiv v_1^5 v_2^3(v_1^2 v_2^3 t_2 + v_1^3 v_2^{-1} t_3^2)$ mod (v_1^9) obtained by (3.4). The third one for $k = 1$ now follows from more calculation $dv_1^4 v_2^{-6} v_3^4 x_1 \equiv v_1^4 v_2^{-6} v_3^4(v_1^2 v_2^3 t_2)$, $dv_1^6 v_2^{-3} v_5 \equiv v_1^6 v_2^{-3}(v_3 t_2^8 - v_3^4 t_2 + v_2 t_3^4 - v_2^8 t_3)$ and $dv_1^6 v_2^{-5} v_3 v_4^2 \equiv v_1^6 v_2^{-5} v_3(v_2^2 t_2^8 - v_2^8 t_2^2)$ all modulo (v_1^7).

Suppose the congruenece for $2k$, which brings $dx_{2k}^2 \equiv v_1^{2a_k} v_2^{2b_k} v_3^{2+2c_k} t_2^4 + v_1^{2a_k} v_2^{4+2b_k} v_3^{2c_k} t_3^2$ mod $(v_1^{2+2a_k})$. We also compute $dv_1^{2a_k} v_2^{2b_k-1} v_3^{2+2c_k} v_4 \equiv v_1^{2a_k} v_2^{2b_k-1} v_3^{2+2c_k}(v_2 t_2^4 - v_2^4 t_2 + v_1 t_3^2)$ mod $(v_1^{2+2a_k})$ to obtain the congruence for $2k+1$.

The inductive hypothesis implies $dx_{2k+1}^2 \equiv v_1^{4a_k} v_2^{4b_k} v_3^{4c_k}(v_2^6 v_3^4 t_2^2 + v_2^8 t_3^4 + v_1^2 v_2^{-2} v_3^4 t_3^4)$ mod $(v_1^{4+4a_k})$. Note here that c_k is a multiple of 4 and so $4c_k$ is of 2^4, which indicates that $dv_3^{4c_k} \equiv 0$ mod (v_1^{16}). We now certify the last one by computing mod (v_1^3)

$$\begin{aligned}
dv_1^{-6}v_2^{10}\,\overline{x_2} &\equiv v_1^{-6}v_2^{10}(v_1^6v_2^{-2}t_3^4 + v_1^8v_2^2t_3^2 + v_1^8v_2^{-2}v_3^2t_2^4)\\
dv_1^{-1}v_2^6v_3^5 &\equiv v_1^{-1}v_2^6v_3^4(v_1t_2^2)\\
dv_1^2v_2^{-3}v_3^4v_5 &\equiv v_1^2v_2^{-3}v_3^4(v_3t_2^8 - v_3^4t_2 + v_2t_3^4 - v_2^8t_3)\\
dv_1^2v_2^{-5}v_3^5v_4^2 &\equiv v_1^2v_2^{-5}v_3^5(v_2^2t_2^8 - v_2^8t_2^2)\\
dv_1^{-10}v_2^2x_3 &\equiv v_1^{-10}v_2^2(v_1^{12}v_2^6(v_2^3v_3^2t_2 + v_2^4t_3^2))\\
dv_1^2v_2^7v_3^2v_4 &\equiv v_1^2v_2^7v_3^2(v_2t_2^4 - v_2^4t_2)\\
dv_2^{-6}v_3^8x_1 &\equiv v_2^{-6}v_3^8(v_1^2v_2^3t_2).
\end{aligned}$$

q.e.d.

In order to study about the δ in the sequence (3.2), we give another representative of h_{31}:

$$h_{31} = t_3^2 + v_1v_3t_2^2 + v_1^4v_3t_2 + v_1^5t_2^3 + v_1^7t_3,$$

which equals to $v_1^{-1}(dv_4 - v_2t_2^4 - v_2^4t_2)$. Since v_1 acts monomorphically on the complex and v_2 and t_2 are primitives, we see

LEMMA 3.5. *$dh_{31} = 0$ in $(BP_*/(2))[t_2, t_3, \cdots]$.*

PROPOSITION 3.6. *The elements v_2, h_{2i} for $i \in Z/2$, and h_{31} are all cycles in the cobar complex Ω_Γ^*C. In the sequence (3.2) δ-images of the elements v_2, h_{2i} and h_{31} are all trivial.*

In the next lemma, we do not write the multiples of v_2's, because we can tell them by virtue of degrees of elements and furthermore they made the formula more complicated though they have no influenece on the module structure which we want to determine.

LEMMA 3.7. *Let δ be the map in the sequence (3.2), then for any elements $\alpha \in K(2)_*[h_{20}] \otimes \Lambda(h_{21}, h_{31})$ and integers $t = 2s+1 > 0$.*

1) $$\delta(x_n^t\alpha/v_1^{A_n}) = \begin{cases} v_3^{2s}h_{21}\alpha & (n=0)\\ v_3^{4s}h_{20}\alpha & (n=1)\\ v_3^{2^{n+1}s+2c_k+2}h_{20}\alpha + v_3^{2^{n+1}s+2c_k}h_{31}\alpha & (n=2k+1,\ k\geq 1)\\ v_3^{2^{n+1}s+c_k+1}h_{21}\alpha + v_3^{2^{n+1}s+c_k}h_{30}\alpha & (n=2k,\ k\geq 1)\end{cases}$$

2) $$\delta(x_n^th_{30}\alpha/v_1) = \begin{cases} v_3^{2s}h_{21}h_{30}\alpha + v_3^th_{20}^2\alpha & (n=0)\\ v_3^{2^nt}h_{20}^2\alpha & (n>0)\end{cases}$$

3) $$\delta(x_n^t\rho\alpha/v_1)=\begin{cases} v_3^{2s}h_{21}\rho\alpha+v_3^t h_{21}h_{31}\alpha+v_2^{t+2}h_{20}h_{21}\alpha & (n=0)\\ v_3^{2^n t}h_{21}h_{31}\alpha+v_3^{2^n t+2}h_{20}h_{21}\alpha & (n>0)\end{cases}$$

Here A_n is an integer a_k if $n=2k$ and $2a_k$ if $n=2k+1$.

PROOF. By virture of Proposition 3.6, we see that

$$\delta(x\alpha)=\delta(x)\alpha,$$

and so it is enough to show that the lemma for $\alpha=1$. Then 1) follows immediately from the definition of δ together with Lemma 3.3. Using Quillen's formula $\sum_{i+j=n}m_i\Delta t_j^{2^i}=\sum_{i+j+k=n}m_i t_j^{2^i}\otimes t_k^{2^{i+j}}$ and Hazewinkel's formula $v_n=pm_n-\sum_{i=1}^{n-1}m_i t_{n-i}^{2^i}$, we compute

$$\Delta t_3=t_3\otimes 1+1\otimes t_3+v_1t_2\otimes t_2$$

$$\Delta t_4=t_4\otimes 1+t_2\otimes t_2^4+1\otimes t_4+v_1t_3\otimes t_3+v_2t_2^2\otimes t_2^2$$

$$\Delta t_5=t_5\otimes 1+t_3\otimes t_2^8+t_2\otimes t_3^4+1\otimes t_4+v_2t_3^2\otimes t_3^2+v_3t_2^4\otimes t_2^4$$

The first formula gives the one

$$\delta(xh_{30}/v_1)=\delta(x/v_1)h_{30}+xh_{20}^2,$$

and by the others together with (3.4) we compute

$$\delta(\rho/v_1)=v_2^3h_{21}h_{31}+v_2^2v_3^2h_{20}h_{21}.$$

Thus we obtain 2) and 3) from Lemma 3.3. q.e.d.

It seems that this enables us to compute all dimension of the groups H^kC, but the complexity of integers appears in the exponent of v_3 prevents us to write down the explicit generators of H^kC. Once we write down them, we conjecture that we need only a few more differentials to compute all groups H^kC. Here we give the first two groups as follows, though we can write down H^kC for more small value k, inductively, by routine computation.

PROPOSITION **3.8.** 1) *H^0C is a tensor product of $K(2)_*$ and a direct sum of $(\boldsymbol{Z}/2[v_1,v_1^{-1}])/(\boldsymbol{Z}/2[v_1])$ and cyclic $\boldsymbol{Z}/2[v_1]$-modules generated by*

$$x_n^t/v_1^{A_n}$$

for $n\geq 0$ and $t=2s+1$ with $s\geq 0$.

2) *H^1C is a tensor product of $K(2)_*$ and a direct sum of $H^0C\otimes\boldsymbol{Z}/2\{h_{31}\}$, $\boldsymbol{Z}/2[v_3]\{h_{30}/v_1,\rho/v_1\}$, and cyclic $\boldsymbol{Z}/2[v_1]$-modules generated by*

$$v_3^{2s+1}h_{20}/v_1,\ h_{20}/v_1,\ v_3^{2t}h_{20}/v_1^2\ (t\neq 0)\ \text{and}\ v_2^th_{21}/v_1$$

for $s\geq 0$ and $t\notin G=\{2^{2k+1}s+c_k+1 \mid k\geq 0\}$.

Recall the exact sequence (3.1)

$$
(3.9) \quad \begin{aligned} 0 &\longrightarrow H^0A \longrightarrow H^0B \longrightarrow H^0C \xrightarrow{\delta} H^1A \longrightarrow H^1B \longrightarrow H^1C \\ &\xrightarrow{\delta} H^2A \longrightarrow H^2B \longrightarrow \cdots. \end{aligned}
$$

In the sequence, the structure of H^*B and H^*C are given in (2.2) and Proposition 3.8.

PROOF OF THEOREM. Since H^*B is isomorphic to $K(1)_*[v_2] \otimes E(h_{20})$ by (2.1) and (2.2), $H^0B = K(1)_*[v_2]$, $H^1B = K(1)_*[v_2]\{h_{20}\}$ *and* $H^kB = 0$ for $k > 1$. Notice that $v_2^4 h_{20}/v_1^2 = h_{31}/v_1$ seen by the second congruence in Lemma 3.3. Then Theorem follows immediately from the exact sequence (3.9). q.e.d.

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