Axiom of Numerical Utility and Conceptual Structure under Uncertain Information by Newmann and Morgenstein

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1. Conceptualizing structure

According to Newmann and Morgenstein, a transformation system is narrowed by the natural operation of the confines of availability, where two availabilities are combined by the following two complementary probabilities:

$$\alpha \tag{1}$$

$$1\text{-}\alpha(0<\alpha<1) \tag{2}$$

Assume the utilities to be "*u*" and "*v*", these utilities are shown in the following natural concern and natural operation, respectively:

$$u>v \tag{3}$$

$$\alpha u+(1\text{-}\alpha)v(0<\alpha<1) \tag{4}$$

(3) means that "*u* is better than *v*" and (4) is the median point of "*u*" and "*v*" which provides "*α,1-α*" at a significance level. Moreover, (3) is the amalgamaion which chooses "*u*" by probability "*α*." Similarly, (4) is the amalgamation which chooses "*v*" by probability "*1-α.*" It is necessary to find the following relation between "utility and numbers," if the measurement in which being of these conceptions and its reappear are possible is accepted:

$$u\rightarrow p=v(u) \tag{5}$$

Where two "*u>v*" (concerning availability) and operation "*α+(1-α) v*" should discover the availability and concerns in the same conception. This relation is assumed in (5), where "*u*" is the utility and "*v (u)*" is a number given by the relation. Newmann and Morgenstein raised the following formulas in this relation:

If $u>v$, then $v(u)>v(v)$ (6)

$$v(\alpha u+(1-\alpha)v)=\alpha v(u)+(1-\alpha)v(v) \tag{7}$$

Suppose the following two relations exist:

$$u\rightarrow\rho=v(u) \tag{8}$$

$$u\rightarrow\rho'=v'(u) \tag{9}$$

Then, (10) is transformed from (8) and (9).

$$\rho\Leftrightarrow\rho' \tag{10}$$

Further, (10) is transformed as follows:

$$\rho'=\varphi(\rho) \tag{11}$$

(8) and (9) replace (6) and (7), respectively. Therefore, the transformation of (10), where "Function $\varphi(\rho)$" retains "Function $\rho>\sigma$" and "Operation $\alpha\rho+(1-\alpha)\sigma$." According to Newmann and Morgenstein, this is described as follows:

If $\rho>\sigma$, then $\varphi(\rho)>\varphi(\sigma)$ (12)

Therefore, "$\varphi(\sigma)$" is transformed into the following linear expressions:

$$\rho'=\varphi(\rho)=\omega0\rho+\omega1 \tag{13}$$

However, "$\omega0,\omega1$" are constant numbers. Moreover, "$\omega0>0$", the mathematical expression of these chains solved the following about utility. If the numerical assessment of availability exists, it will settle, except in linear expression transformations. Considering this logic, the existence of numerical transformation will provide clarity, if the following are solved: "related $u>v$" and "operation $\alpha u+(1-\alpha)\ v$" axiomatized.

2. Option and interpretation of the system of axioms

According to Newmann and Morgenstein, an axiom is needed to attain a specific purpose. Moreover, an axiom should be as simple as possible and should include intuitive fairness.

Newmann and Morgenstein interpreted mathematical axioms and attempted to evaluate the conception by clarifying the correspondence of "(6) and (7)→(8)." However, the axiom is formulized by one in order to attain intuitive and aesthetic causes.

The system of axioms of Newmann and Morgenstein is as follows. The "system *U*" exists temporarily. The components of "system *U*" are "*u, v, w,...*" *U* contains "related $u>v$" and the arbitrary numbers "α $(0< \alpha<1)$." In that case, the following operation is given to *U*:

$$\alpha u+(1-\alpha) v=w \tag{14}$$

Furthermore, it is assumed that (14) fulfills the following axioms (Newmann and Morgenstein): "$u>v$ is all the order relation in *U*." Considering (15), only one is materialized among the following three concerns:

$$u>v,\ u<v, \text{ and } u=v \tag{15}$$

The result will be "$u >w$" if $v>w$ is "$u> v$."

Moreover, order relation is expressed by the following joint operation:

$$\text{If "}u<v\text{", then "}u<\alpha u+(1-\alpha) v\text{"} \tag{16}$$

$$\text{If "}u >v\text{", then "}u >\alpha u+(1-\alpha) v\text{"} \tag{17}$$

$$\text{In "}u >w> v\text{," } \alpha \text{ which replaces "}w>\alpha u+(1-\alpha) v\text{" exists} \tag{18}$$

Further, the joint operation is as follows:

$$\text{"}\alpha u+(1-\alpha) v = v+\alpha(1-\alpha) u\text{"} \tag{19}$$

$$\alpha \quad (\beta u+(1-\beta))+(1-\alpha)v=\gamma u +(1-\gamma)v \tag{20}$$

$$\text{However, } \gamma=\alpha\beta \tag{21}$$

These axioms specify "(6) and (7)→(8)." Therefore, the conclusion of "1" is applied to *U*, where in this section, *U* is a system of abstract availability. However, it serves as "a number of systems decided except for primary type transformation" (Newmann and Morgenstein).

3. General theory about the system of axioms

Newmann and Morgenstein have described the axiomatic account of utility and I described the preceding clause that "(6) and (7) draw the numerical personality of availability." (7) can combine the figure of availability, such as a mathematical expectation (probability) (Newmann and Morgenstein). However, the mathematical expectation as a conception presents interrogatory, where it is dependent on the fairness of the conception on a certain "assumption" about the personality of "expectancy." In short, in a case such as a bet, the availability of the act is incompatible with a mathematical expectation. According to Newmann and Morgenstein, the system of axioms "(15)-(19), (20), and (21)" avoid the capability of the availability of a bet.

Moreover, according to them, this system of axioms does not avoid this matter intentionally. As a result, axioms (20) and (21) are the same and use a mathematical expectation. Furthermore, the numerical availability of fulfilling this axiom is drawn by a system of axioms "(15)-(19), (20), and (21)." This defines the mathematical availability which can actually apply the calculation of mathematical expectation. The conceptual constitution required for this is guaranteed from a system of axioms "(15)-(19), (20), and (21)."

The system of axioms of Newmann and Morgenstein is based on the relation and operation of availability. In this case, operation is provided more directly than the relation of availability Because the person who can imagine two availability "$(u,\ v)$" Status is respectively fixed Probability "$(\alpha, 1\text{-}\alpha)$", and it has these two Status simultaneously. On the other hand, the assumption produced by the axiom about "$u>v$," (completeness of this order), may have misdoubts. For example, any case with the availability "u and v" could not be determined correctly. However, even if this misdoubt has a big value, this capability (completeness of the individual preference structure) must be assumed in the indifference curve analysis. However, if this personality (related to "$u>v$") is assumed, numerical availability will be acquired by the operation "$(\alpha u+(1\text{-}\alpha)v$" which is a marginally doubtful point (Newmann and Morgenstein).

In addition to this, Newmann and Morgenstein mentioned the following important matters. First, they made availability to one person object of the research. That is, these consideration does not derive the availability of other person differs. Second, their research is inapplicable to the analysis of a mathematical expectation.

4. Role of a final utility concept

Considering the above-mentioned issue, Newmann and Morgenstein specified the efficiency of a numeric availability concept. Moreover, in an objective status, an economic agent assumes that material conditions are known (Newmann and Morgenstein). All the economic agents perform statistical and mathematical operations under such a status. This is premised on full knowledge of the material conditions by an economic agent and can be set objectively. This important nature has been highlighted by several researches. However, this review is insufficient.

Newmann and Morgenstein do not consider this issue in depth. Rather, their consideration assumes the "information perfection nature" on the material conditions of the objective status. Generally, economic and social phenomena that depend on the individual "information imperfection nature" appear in their rationale. Thus, their rationale assumes the "perfect nature of information" and it is stated that these phenomena are not related to individual "imperfections of information." A remarkable example of this is shown by "discrimination," "tolerant plundering," "accommodation," and "transfer" (Newmann and Morgenstein). Furthermore, they made an exception to "information imperfection" by seriously considering the phenomenon resulting from the factor that serves as a case where it is absolutely not related.

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