Household Water Consumption Model and Its Application for Evaluation of Emergency Water Supply Measures under Suspension of Water Supply

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Abstract: The household behavioral model about water use activities under the condition of a suspension of water supply was examined. Water is supplied at emergency water supply stations and people obtain water there and have to carry to their homes. There are constraints in available water and time. We assume that the household maximizes its utility by inputting water using activities, leisure and other composite commodities. The household's water using activities were modeled such that the household produces water-dependent domestic services and consumes them by itself. Using the obtained model the behavior of the household which was damaged by 2001 Geiyo earthquake occurred in Japan was investigated. The way to estimate benefit so as to make distance to carry water shorter was examined.

Key words: domestic water consumption, time allocation, emergency water supply, earthquake countermeasures

1. INTRODUCTION

When an earthquake occurs and water supply networks are broken and cause a suspension of water supply, emergency water supply is performed. The temporal water supply stations are constructed, from which people can obtain water and carry to their home. People try to obtain enough water to do their daily water consuming activities such as toilet flushing and cloths washing. However, carrying enough amount of water to their homes takes much time and labor. Therefore, the density of emergency water supply stations has a great effect on the amount of water which people can use. The waterworks undertaking should take a notice to the situation of people who obtain water when it makes a plan of emergency water supply stations. Those have been generally set up at public facilities such as schools without any thought of consumers' convenience. While water volume to supply for people has been examined, the distance that consumers have to carry has not been fairly considered. Even if the supplier prepare sufficient water, residents cannot use satisfactorily in the case that distance is too long to carry whole planned water.

In this study the way to establish emergency water supply stations was examined from the point of consumers' convenience. Behavioral model about water use of a household under the restricted condition that a water supply system was destructed by an earthquake and water is supplied from an emergency water supply station was considered. Using the developed model the benefit of emergency water supply station was evaluated.

2. WATER USE BEHAVIORAL MODEL OF HOUSEHOLD

We assume that a household produces water-dependent domestic services such as kitchen work, clothes washing and so on using water and consume the services by itself (Okada et al. [1]). At the time of a suspension of water supply, households allocate their limited resources to obtain water and produce water-dependent domestic services as well as to do other activities and obtain commodities. A portion of water is substituted by time and/or some commodities depending on those prices. For example, clothes are washed by hand in stead of a washing machine when water is not enough to use the machine. It takes more time, that is, shortage water is substituted by time to wash clothes.

2.1. Utility Maximization

Households are considered to consume total water-dependent domestic services X, leisure L and composite commodity G so as to maximize their utility under the constraint of the total income. It is written as follows,

$$V(P_X, P_L, I) = \max_{X, L, G} \{U(X, L, G\} \tag{1}$$

subject to $P_X X + P_L L + G = I$ (2)

where $U(X,L,G)$ is utility function. P_X and P_L are prices of water and leisure, respectively. G is assumed to be numeraire. I is the total income and V is indirect utility function.

Time constraint is written as follows

$$T_X + L = T_0 - T_W \tag{3}$$

where T_X, T_W and T_0 are time for water using activities, working time and total available time, respectively. T_X is given by time used for producing water-dependent domestic services and obtaining water as follows,

$$T_X = \sum_i t_i + \tau Q \tag{4}$$

t_i is time used for producing water-dependent domestic service i. τ is time necessary for obtaining unit volume of water. Q is total volume of water given by

$$Q = \sum_i q_i \tag{5}$$

where q_i is water used to produce water-dependent domestic service i.

Budget constraint is given by

$$w_1'Q + w_3 \sum_i g_i + G = y + w_2 T_W \tag{6}$$

w_1' is cost necessary for obtaining water from the water supply station such as water carriers. w_2 is wage rate. w_3 is price of market goods used for producing water-dependent domestic services. y is income.

When time for labor is substituted by income with wage rate w_2, the following equation is obtained from Eq.(3) and Eq.(6)

$$(w_1' + w_2\tau)Q + G + w_2 \sum_i t_i + w_3 \sum_i g_i + w_2 L + G = y + w_2 T_0 = I \tag{7}$$

It is rewritten as

$$C_X + C_L + G = P_X + w_2 L + G = I \tag{8}$$

C_X is the price of the total water-dependent service described by

$$C_X = X \quad P_X = w_1 Q + w_2 T_X + w_3 G_X \tag{9}$$

where w_1 is the generalized price of water given by $w_1 = w_1' + w_2\tau$, T_X is the total amount of used water given by $T_X = \Sigma t_i$ and G_X is total unit of commodities used for water using activities given by $G_X = \Sigma g_i$.

2.2. Production of water-dependent service

Total water-dependent domestic service is produced with some individual water-dependent domestic services z_i as shown in Figure 1. The price of the total water-dependent domestic service is given by the cost function which is obtained by minimizing the total cost of individual water-dependent domestic service with the constraint of production of the total water-dependent domestic service as X. It is mathematically stated as follows,

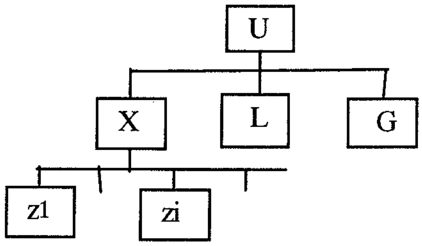

Fig. 1 Model structure

$$C_X = \min_{z_i} \sum_i p_i z_i \tag{10}$$

$$\text{subject to } X = X(z_i) \tag{11}$$

$X(z_i)$ is production function.

We consider that each water-dependent domestic service is produced with water, time and market goods.

$$c_i = \min_{q_i, t_i, g_i} \{w_1 q_i + w_2 t_i + w_3 g_i\} \tag{12}$$

$$\text{subject to } z_i = f_i(q_i, t_i, g_i) \tag{13}$$

where c_i is the cost function of z_i, f_i is the production function. q_i, t_i and g_i are water, time and market goods, respectively. The price of water-dependent domestic service z_i in Eq.(10) is given by the cost function as,

$$p_i z_i = c_i \tag{14}$$

2.3. Benefit estimation of improvement of emergency water supply environment

When the emergency water supply environment is improved such that the distance between a water supply station and a home is reduced, the generalized price of water becomes small and it affects total utility of a household. The derivative of the indirect utility function is given as follows,

$$dV = \frac{\partial V}{\partial P_X} dP_X + \frac{\partial V}{\partial P_L} dP_L + \frac{\partial V}{\partial I} dI \tag{15}$$

Using Roy's Identity and the Lagrangian multiplier λ introduced to solve the utility maximization problem which is equivalent to marginal utility of income, the above equation is rewritten as follows,

$$\begin{aligned} dV &= \frac{\partial V}{\partial P_X} dP_X + \frac{\partial V}{\partial P_L} dP_L + \frac{\partial V}{\partial I} dI \\ &= -X_0(P_X, P_L, I)\frac{\partial V}{\partial I} dP_X - L_0(P_X, P_L, I)\frac{\partial V}{\partial I} dP_L + \frac{\partial V}{\partial I} dI \\ &= -\lambda X_0(P_X, P_L, I) dP_X - \lambda L_0(P_X, P_L, I) dP_L + \lambda dI \end{aligned} \tag{16}$$

in which X_0 and L_0 are the ordinary demand functions of the total water-dependent domestic service and leisure, respectively. Therefore, the change of utility is given by the price base as follows,

$$\frac{dV}{\lambda} = -X_0(P_X, P_L, I) dP_X - L_0(P_X, P_L, I) dP_L + dI \tag{17}$$

If the change of the generalized water price is assumed no to affect the wage rate and the total income ,that is, partial equilibrium is assumed, the second and third terms of the right side of the above equation are neglected. When the price of the total water-dependent domestic service is changed from P_{X1} to P_{X2}, the change of utility is equivalent to the change of consumer's surplus ΔCS and they are written as follows,

$$\Delta CS = \int_{P_{x1}}^{P_{x2}} \frac{dV}{\lambda} = -\int_{P_{x1}}^{P_{x2}} X_0(P_x, P_L, I)\, dP_x \tag{18}$$

2.4. Examination by assuming types of functions

In order to examine the model, for the simplicity, we assume that the water-dependent domestic service is produced by using water and time and it is described by the Cobb-Douglas type production function.
Eqs.(12) and (13) are rewritten as follows,

$$c_i = \min_{q_i, t_i} \{w_1 q_i + w_2 t_i\} \tag{19}$$

subject to $z_i = A_i q_i^{\alpha_i} t_i^{1-\alpha_i}$ (20)

The conditional demands of factor of production, the cost function and the price function, respectively, are obtained as follows,.

$$q_i = \frac{1}{A_i}\left(\frac{w_1}{\alpha_i}\right)^{\alpha_i}\left(\frac{w_2}{1-\alpha_i}\right)^{1-\alpha_i}\frac{\alpha_i}{w_1} z_i \tag{21}$$

$$t_i = \frac{1}{A_i}\left(\frac{w_1}{\alpha_i}\right)^{\alpha_i}\left(\frac{w_2}{1-\alpha_i}\right)^{1-\alpha_i}\frac{1-\alpha_i}{w_2} z_i \tag{22}$$

$$c_i = \frac{1}{A_i}\left(\frac{w_1}{\alpha_i}\right)^{\alpha_i}\left(\frac{w_2}{1-\alpha_i}\right)^{1-\alpha_i} z_i \tag{23}$$

$$p_i = \frac{1}{A_i}\left(\frac{w_1}{\alpha_i}\right)^{\alpha_i}\left(\frac{w_2}{1-\alpha_i}\right)^{1-\alpha_i} \tag{24}$$

The total water-dependent domestic service is produced by composing the individual water-dependent domestic services. The substitution between the individual water-dependent domestic services is thought to be difficult such as between toilet flushing and kitchen work. Therefore, the Leontief type production function is assumed for the total water-dependent domestic service. Thus Eq.(11) is written as follows,

$$X = \min \{b_i z_i\} \tag{25}$$

Then the conditional demand of factor of production is obtained as follows,

$$z_i = \frac{X}{b_i} \tag{26}$$

The cost function and the price of X are obtained and rewritten by using Eq.(24) as follows,

$$C_x = \sum_i \frac{p_i}{b_i} X = \sum_i \frac{1}{A_i}\left(\frac{w_1}{\alpha_i}\right)^{\alpha_i}\left(\frac{w_2}{1-\alpha_i}\right)^{1-\alpha_i} X \tag{27}$$

$$P_x = \sum_i \frac{p_i}{b_i} = \sum_i \frac{1}{A_i}\left(\frac{w_1}{\alpha_i}\right)^{\alpha_i}\left(\frac{w_2}{1-\alpha_i}\right)^{1-\alpha_i} \tag{28}$$

Finally we assume that the household utility function is described by the Cobb-Douglas type. Then Eq.(1) is rewritten as follows,

$$V(P_X, P_L, I) = \max_{X,L,G} \{U(X, L, G\} = \max_{X,L,G} K\left(\frac{\alpha}{P_X}\right)^{\alpha}\left(\frac{\beta}{P_L}\right)^{\beta}(1-\alpha-\beta)^{1-\alpha-\beta} I \tag{29}$$

From the solution of utility maximization under the constraint of Eq.(2), the ordinary demand function is obtained as

$$X_0 = \frac{\alpha}{P_X} I, \quad L_0 = \frac{\beta}{P_L} I, \quad G = (1-\alpha-\beta)I \tag{30}$$

Substituting Eq.(30) for Eq.(18), the consumer's surplus is obtained as follows,

$$\Delta CS = -\int_{P_{X1}}^{P_{X2}} \frac{\alpha}{P_X} I \, dP_X \tag{31}$$

3. CASE STUDY

3.1 Parameter setting

The survey about household water use under the condition of a suspension of water supply was carried out in Yutaka town Hiroshima prefecture Japan (Hosoi and Masuda [2]). The town was attacked by 2001 Geiyo Earthquake. The water transportation pipe was broken and water supply stopped for 3 days. Emergency water supply was carried out by setting the emergency supply stations during those days.

The time used to obtain water, the volumes of obtained water, used water for water-dependent domestic services and additional time used for water-dependent domestic service to ordinary cases were surveyed. The activity of the average household was estimated as shown in the left three columns of Table 1. The obtained water was once saved in containers at home and had to be carried to the place of use. This time was included in the time used for production of a water-dependent domestic service. According to the survey bath was not used. A washing machine was also not used and clothes were washed by hand.

Table 1 The determined values of the parameters of water-dependent domestic services

Water-dependent services	Water (L per person per day)	Time (minutes per person per day)	αi	Cost (dollars)
Toilet	13.5	37.8	0.19	0.78
Kitchen	25.3	30.8	0.35	0.79
Clothes washing	19.0	24.3	0.34	0.61
Total	57.8	92.9		2.18

The number of family of the average household was 2.4 people, 90 minutes was necessary to obtain total volume of water which is 138.7 L, that is, 57.8 L for each. If the wage rate is assumed to be 10 dollars for an hour, the price of water is estimated as 0.108 dollars/L. Therefore α_i of Eq.(20) is calculated as shown in the forth column of Table 1. The costs of the water-dependent domestic services in the surveyed average household are shown in the fifth column of Table 1.

After the earthquake each parson consumed 4.7 hours a day to put the damaged home in order. If sleeping time is assumed to be 8 hours, total available time is 11.3 hours a day. Therefore total income is estimated as 113 dollars a day. The mean labor time was estimated 5.5 hours a day and the time used for water-dependent domestic service production was 92.9 minutes as indicated in Table 1. The remained time is used for leisure. Therefore the budgets used for the production of the total water-dependent domestic service, leisure and composite commodities are estimated as shown in

Table 2. Using those values α and β of Eq.(29) are calculated as shown in Table 2.

Using the determined values, the change of water volume and time to produce water-dependent services following to the change of the generalized water price are examined and shown in Figure 2. As the price of water increase, water used to produce each water-dependent domestic service decreases. Time to be used is, however, shows different tendency depending on the service. For the toilet, time decreases with water. To the contrast it increases in kitchen and clothes washing. In the toilet the decrease in water directly relates to the decrease in time. Time can substitute water in kitchen and clothes washing.

Table 2 The determined values of the utility function

Input	Cost (dollars)	
Total water-dependent service	21.73	α=0.192
Leisure	36.43	β=0.322
Composite commodity	54.85	1-α-β=0.485
Total	113	

3.2. Evaluation of effect of improvement of emergency water supply environment

If the total water-dependent domestic service is produced 1 unit under the current condition, its price is estimated 2.18 dollars which is obtained as the total cost of individual water-dependent domestic services shown in Table 1. Figure 3 shows the relationship between w_1 and P_X. The change of generalized water price affects the price of the total water-dependent domestic service. If the emergency water supply stations are increased and necessary time to obtain water is reduced from current 90 minutes to the half namely 45 minutes, the generalized price of water becomes 0.054 dollars that is half of the current price. The price of the total water-dependent domestic service changes from 21.76 dollars to 17.86 dollars. The total income is 113 dollars and α is 0.192 as shown in Table 2. Therefore substituting these values for Eq.(31) the benefit of this improvement is calculated as follows,

$$\Delta CS = -\int_{21.76}^{17.86} \frac{0.192 \times 113}{P_X} dPX = 4.29$$

The benefit is estimated as 4.29 dollars a day for each parson.

Figure 4 demonstrates the demand functions X_0 when the total income changes. If other time does not change, I=\$140 means that the time to put damaged home in order is 2 hours, that is, the damage is smaller than the current case study. When I=\$100, the time to put damaged home in order is 8 hours, that means the damage is larger than the current case study. As we can see from Figure 4, if the improvement of the emergency water supply environment,

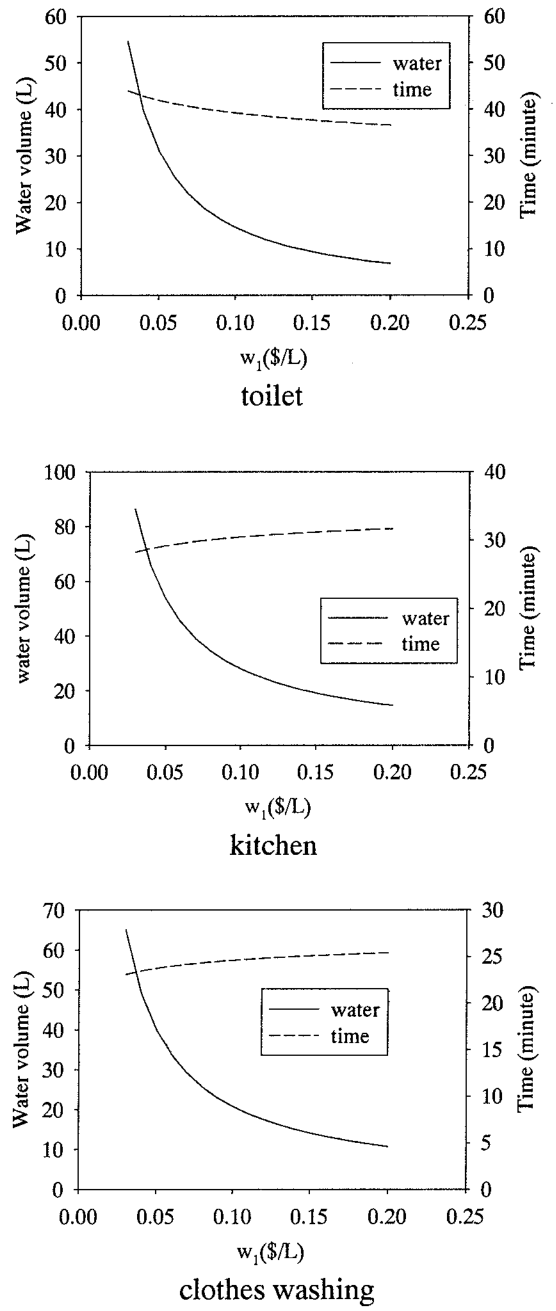

Fig. 2 The relationships between the generalized water price and demand function of factor of production of water dependent-services

that is, the reduction of P_X, is same, larger consumer's surplus appears at smaller damaged household. For example ΔCS is calculated as 5.31 dollars in the case of I=\$140 and 3.79 dollars in I=\$100 with the same values of other parameters as the case study. Therefore, more stations are necessary to bring same benefit for the area suffered from larger earthquake which brings larger damage in order to bring equal welfare.

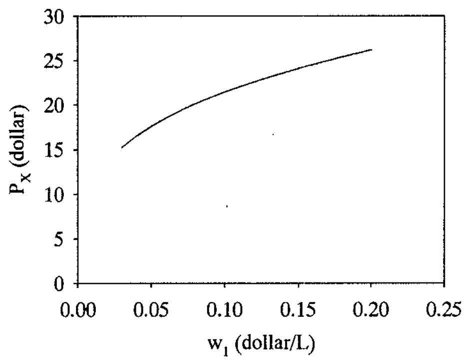

Fig. 3 The relationship between the generalized water price and the price of the total water-dependence domestic service

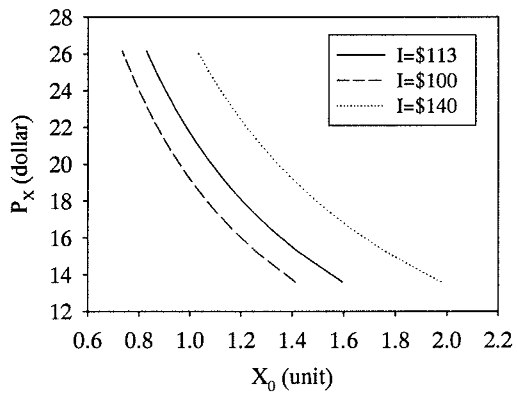

Fig. 4 Demand function of the total water-dependent domestic service

4. CONCLUDING REMARKS

The household behavior model considering water consuming activities under the condition of a suspension of water supply was investigated. It can show one of the way of cost benefit analysis of counter measures to disaster damages of a water supply system. The case study treated only the simple case that includes toilet flushing, kitchen and clothes washing as water consuming activities and only water and time are considered as factors of production of water using domestic services. We will improve the model by collecting more data.

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