Stability Robustness Analysis for Interval Systems in Frequency Domain

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Abstract: In this paper, we describe modeling and composition rules of frequency response characteristics based on experimental data of plants (controlled systems) with uncertainty and nonlinearity, and the robust stability evaluation of feedback control systems. Analysis and design of control systems using the upper and lower bounds of such experimental data would be effective as a practicable method which is not heavily dependent upon mathematical model such as the transfer function. In this report, sets of the experimental data are assumed to include not only the band of the gain characteristics but also the band of the phase characteristics. The stability robustness of the feedback control system is investigated based on modeling and composition rules of the interval of frequency response characteristics. Numerical examples are shown to illustrate the stability robustness for interval systems.

Key words: Frequency response, interval system, Kharitonov theorem, robust stability, uncertainty, nonlinearity.

1. INTRODUCTION

The controlled systems in practice should be modeled in the frequency domain, considering physical characteristics and uncertainties in the high-frequency range. When considering the physical characteristics, the model should be determined based on the frequency response characteristics of the input and the output of the controlled system.

In this paper, sets of the experimental data are assumed to include not only the band of the gain characteristic but also the band of the phase characteristic [1]. The stability robustness of the feedback control system is investigated based on modeling and composition rules of such a frequency response characteristic [2][3][4]. By taking phase characteristic into consideration, gain change and sector nonlinearity can be included in the same discussion.

2. FREQUENCY RESPONSE INTERVAL

Consider a controlled element with uncertainty by the multiplicative perturbation as shown in Fig. 1, that is,

$$G^*(j\omega) = G(j\omega)(1 + \Delta(j\omega)). \qquad (1)$$

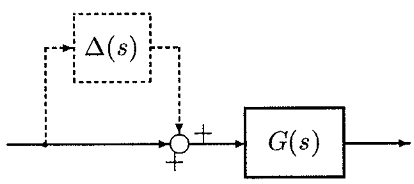

Fig. 1 Control element with multiplicative perturbation.

The frequency response characteristic of uncertain term

$$\Delta(j\omega) = \frac{G^*(j\omega) - G(j\omega)}{G(j\omega)}$$

is assumed to be given by not only the upper and lower bounds of the absolute value, but also by the interval of the argument determined theoretically or experimentally.

In addition, the absolute value and the argument of the frequency response characteristic of the uncertain term

$$\Delta(j\omega) = |\Delta(j\omega)|e^{j\angle\Delta(j\omega)} = \alpha(\omega) + j\beta(\omega) \quad (2)$$

are expressed respectively as the following interval:

$$|\Delta(j\omega)| = r(\omega) \in [\underline{r}(\omega); \overline{r}(\omega)], \quad (3)$$

$$\angle\Delta(j\omega) = \phi(\omega) \in [\underline{\phi}(\omega); \overline{\phi}(\omega)]. \quad (4)$$

Although the band of such frequency response characteristics in an actual system contains various errors during actual experiments, that is, identification errors, it is remarkably influenced by nonlinear characteristics of each controlled element.

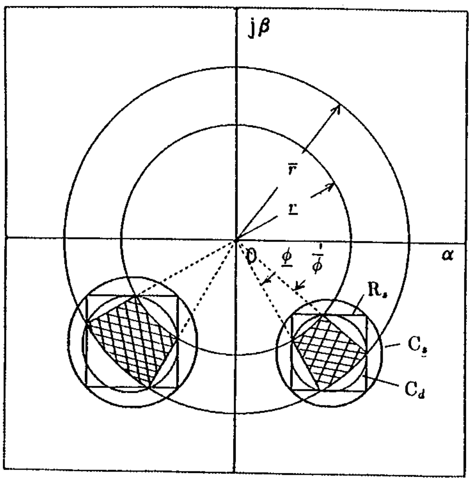

Fig. 2 Frequency response characteristics area for fixed $\omega = \omega_0$.

When the frequency is temporarily frozen with fixed $\omega = \omega_0 \geq 0$ as shown in Fig. 2, the interval of real part α and imaginary part β can be written respectively as follows:

$$\mathcal{A}(\omega_0) = [\underline{\alpha}(\omega_0); \overline{\alpha}(\omega_0)], \quad \mathcal{B}(\omega_0) = [\underline{\beta}(\omega_0); \overline{\beta}(\omega_0)], \quad (5)$$

where

$$\begin{aligned} \underline{\alpha} &= \min\{\, \underline{r}\cos\underline{\phi},\ \underline{r}\cos\overline{\phi},\ \overline{r}\cos\underline{\phi},\ \overline{r}\cos\overline{\phi}\,\}, \\ \overline{\alpha} &= \max\{\, \overline{r}\cos\underline{\phi},\ \overline{r}\cos\overline{\phi},\ \underline{r}\cos\underline{\phi},\ \underline{r}\cos\overline{\phi}\,\}, \\ \underline{\beta} &= \min\{\, \underline{r}\sin\underline{\phi},\ \underline{r}\sin\overline{\phi},\ \overline{r}\sin\underline{\phi},\ \overline{r}\sin\overline{\phi}\,\}, \\ \overline{\beta} &= \max\{\, \overline{r}\sin\underline{\phi},\ \overline{r}\sin\overline{\phi},\ \underline{r}\sin\underline{\phi},\ \underline{r}\sin\overline{\phi}\,\}. \end{aligned}$$

Here, in the case of $(\underline{\phi} + \frac{n\pi}{2})(\overline{\phi} + \frac{n\pi}{2}) < 0$, the upper and lower bounds of α, β are rewritten as follows:

$$\underline{\alpha} = -\overline{r}, \ (n = 2, 6, \cdots), \quad \overline{\alpha} = \overline{r}, \ (n = 0, 4, \cdots),$$
$$\underline{\beta} = -\overline{r}, \ (n = 3, 7, \cdots), \quad \overline{\beta} = \overline{r}, \ (n = 1, 5, \cdots).$$

As an extreme example, the case where phase shift is zero, that is, $\phi \to 0$ is also included in the above discussion. This case corresponds to a sector nonlinear characteristic and an interval set parameter.

Rectangular area R_s is determined by interval sets Eq. (5) from the sectorial area as shown in Fig. 2. The center of rectangle is obviously written as follows:

$$\alpha_c(\omega_0) = \frac{\overline{\alpha}(\omega_0) + \underline{\alpha}(\omega_0)}{2}, \quad \beta_c(\omega_0) = \frac{\overline{\beta}(\omega_0) + \underline{\beta}(\omega_0)}{2}. \quad (6)$$

Thus, rectangular R_s is also covered with circle C_s represented by

center : (α_c, β_c),

radius : $r_c = \dfrac{\sqrt{(\overline{\alpha} - \underline{\alpha})^2 + (\overline{\beta} - \underline{\beta})^2}}{2}$.

In general, however, the area can also be covered with a smaller circle C_d as shown in the figure [1]. The center of the circle is on the straight line (radial line) of phase angle

$$\phi_0 = \frac{(\overline{\phi} + \underline{\phi})}{2}$$

and the circle passes the following four points:

$$p_1 = \underline{r}e^{j\underline{\phi}}, \quad p_2 = \underline{r}e^{j\overline{\phi}}, \quad p_3 = \overline{r}e^{j\underline{\phi}}, \quad p_4 = \overline{r}e^{j\overline{\phi}}.$$

[Theorem 1] The circle which passes above four points p_1, p_2, p_3, p_4 is represented by

center : (α_d, β_d),

radius : $r_d = \sqrt{\underline{r}^2 + r_0^2 - 2\underline{r}r_0\cos(\overline{\phi} - \phi_0)}$
$= \sqrt{\overline{r}^2 + r_0^2 - 2\overline{r}r_0\cos(\overline{\phi} - \phi_0)}$,

where

$$\alpha_d = r_0\cos\phi_0, \quad \beta_d = r_0\sin\phi_0,$$
$$r_0 = \frac{\overline{r} + \underline{r}}{2\cos(\frac{\overline{\phi} - \underline{\phi}}{2})}.$$

(Proof) Due to its symmetry, radius r_0 on the radial line of angle ϕ_0 which is at an equal distance from points p_1 and p_2 have only to be determined. That is,

$$r_d = |\underline{r}e^{j\overline{\phi}} - r_0 e^{j\phi_0}| = |\overline{r}e^{j\overline{\phi}} - r_0 e^{j\phi_0}| \quad (7)$$

From this equation,

$$\overline{r}^2 - \underline{r}^2 - 2r_0(\overline{r} - \underline{r})\cos(\overline{\phi} - \phi_0) = 0. \quad (8)$$

When $\overline{r} \neq \underline{r}$,

$$\overline{r} + \underline{r} - 2r_0\cos(\overline{\phi} - \phi_0) = 0.$$

Therefore,

$$r_0 = \frac{\overline{r} + \underline{r}}{2\cos(\overline{\phi} - \phi_0)}. \quad (9)$$

As for r_0 in Eq. (9), the square of radius r_d can be given as follows:

$$\begin{aligned} r_d^2 &= \underline{r}^2 + r_0^2 - 2\underline{r}r_0\cos(\overline{\phi} - \phi_0) \\ &= \overline{r}^2 + r_0^2 - 2\overline{r}r_0\cos(\overline{\phi} - \phi_0). \quad (10) \end{aligned}$$

□

3. MODIFICATION OF NOMINAL SYSTEM

When the center of a sectorial area of $\Delta(j\omega_0)$ is off the origin as shown in Fig. 2, correcting the nominal system may facilitate a subsequent analysis. There are various approaches in modification of the nominal system. One of the methods is taking the frequency response characteristic at rectangular center (α_c, β_c), the center of circle C_s. Another is a method of taking the frequency response characteristic of the nominal system at (α_d, β_d), the center of circle C_d.

In the manners described above, the modified system can be rewritten as follows:

$$G^* = G(G_c + \Delta_c) = G_m(1 + \Delta_m), \quad (11)$$

where G_m and Δ_m are expressed as

$$\begin{aligned} G_m &= G \cdot G_c, \quad G_c = 1 + \alpha_c + j\beta_c, \quad (12) \\ \Delta_m &= \alpha_m + j\beta_m = \frac{\Delta_c}{G_c} \quad (13) \end{aligned}$$

for the case of circle C_s. In these expressions, variable ω is abbreviated.

Obviously, interval sets α_m, β_m can be written as the following symmetric expressions:

$$\mathcal{A}_m = [-\overline{\alpha}_m; \overline{\alpha}_m], \quad \mathcal{B}_m = [-\overline{\beta}_m; \overline{\beta}_m],$$

where $\overline{\alpha}_m \geq 0$ and $\overline{\beta}_m \geq 0$. The radius r_m of the circle which covers this area is expressed as

$$r_m(\omega) = \frac{r_c(\omega)}{|G_c(j\omega)|}. \quad (14)$$

In the above discussion, it should be noted that the modified nominal system,

$$G_m(j\omega) = G(j\omega)G_c(j\omega) \quad (15)$$

might not be a parameterized frequency response, because $G_c(j\omega)$ is not generally parameterized.

4. COMPOSITION OF FREQUENCY RESPONSE

In this section, $G(s)$ and $\Delta(s)$ are identified with $G_m(s)$ and $\Delta_m(s)$, respectively. The transfer function of the cascade-connected systems is written as

$$G^*(j\omega) = G(j\omega)(1 + \Delta(j\omega)), \quad (16)$$

where

$$\begin{aligned} G(j\omega) &= G_1(j\omega)G_2(j\omega), \quad (17) \\ \Delta(j\omega) &= (1 + \Delta_2(j\omega))\Delta_1(j\omega) + \Delta_2(j\omega). \quad (18) \end{aligned}$$

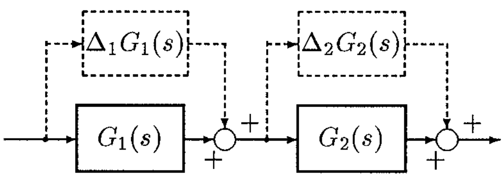

Fig. 3 Cascade-connected plants.

[Theorem 2] As for the absolute value of the frequency response of the uncertain term (radius), the following inequality can be given [3]:

$$\begin{aligned} &|\Delta(j\omega)| \leq |1 + \Delta_2(j\omega)| \cdot |\Delta_1(j\omega)| + |\Delta_2(j\omega)| \\ &\leq |\Delta_1(j\omega)| + |\Delta_2(j\omega)| + |\Delta_1(j\omega)| \cdot |\Delta_2(j\omega)|. \quad (19) \end{aligned}$$

(Proof) Eq. (19) can be easily obtained from Eq. (18) by using the triangular inequality in the complex plane. □

When information on the interval set of phase characteristics or the intervals of a real and an imaginary part is given by

$$\Delta_i(j\omega) = \alpha_i(\omega) + j\beta_i(\omega), \quad i = 1, 2 \quad (20)$$

where

$$\alpha_i \in \mathcal{A}_i = [\underline{\alpha}_i;\ \overline{\alpha}_i],\quad \beta_i \in \mathcal{B}_i = [\underline{\beta}_i;\ \overline{\beta}_i],$$

the following theorem is obtained.

[Theorem 3] The cascade connection (composition) of the uncertain terms represented by Eqs. (16) and (20) based on the multiplicative perturbation is written as

$$\begin{aligned} \Delta(j\omega) &= \alpha(\omega) + j\beta(\omega), \qquad (21)\\ \alpha \in \mathcal{A} &= [\underline{\alpha};\ \overline{\alpha}],\quad \beta \in \mathcal{B} = [\underline{\beta};\ \overline{\beta}], \end{aligned}$$

and

$$\begin{aligned} \overline{\alpha} &= \overline{\alpha}_1 + \overline{\alpha}_2 + \max\{\ \underline{\alpha}_1\underline{\alpha}_2, \underline{\alpha}_1\overline{\alpha}_2, \overline{\alpha}_1\underline{\alpha}_2, \overline{\alpha}_1\overline{\alpha}_2\ \}\\ &- \min\{\ \underline{\beta}_1\underline{\beta}_2, \underline{\beta}_1\overline{\beta}_2, \overline{\beta}_1\underline{\beta}_2, \overline{\beta}_1\overline{\beta}_2\},\\ \underline{\alpha} &= \underline{\alpha}_1 + \underline{\alpha}_2 + \min\{\ \underline{\alpha}_1\underline{\alpha}_2, \underline{\alpha}_1\overline{\alpha}_2, \overline{\alpha}_1\underline{\alpha}_2, \overline{\alpha}_1\overline{\alpha}_2\ \}\\ &- \max\{\ \underline{\beta}_1\underline{\beta}_2, \underline{\beta}_1\overline{\beta}_2, \overline{\beta}_1\underline{\beta}_2, \overline{\beta}_1\overline{\beta}_2\ \},\\ \overline{\beta} &= \overline{\beta}_1 + \overline{\beta}_2 + \max\{\ \underline{\beta}_1\underline{\alpha}_2, \underline{\beta}_1\overline{\alpha}_2, \overline{\beta}_1\underline{\alpha}_2, \overline{\beta}_1\overline{\alpha}_2\ \}\\ &+ \max\{\ \underline{\alpha}_1\underline{\beta}_2, \underline{\alpha}_1\overline{\beta}_2, \overline{\alpha}_1\underline{\beta}_2, \overline{\alpha}_1\overline{\beta}_2\ \},\\ \underline{\beta} &= \underline{\beta}_1 + \underline{\beta}_2 + \min\{\ \underline{\beta}_1\underline{\alpha}_2, \underline{\beta}_1\overline{\alpha}_2, \overline{\beta}_1\underline{\alpha}_2, \overline{\beta}_1\overline{\alpha}_2\ \}\\ &+ \min\{\underline{\alpha}_1\underline{\beta}_2, \underline{\alpha}_1\overline{\beta}_2, \overline{\alpha}_1\underline{\beta}_2, \overline{\alpha}_1\overline{\beta}_2\ \}. \end{aligned}$$

(Proof) The interval set representation of Eq. (21) is expressed as

$$\begin{aligned} \Delta(j\omega) &\in [\underline{\alpha}(\omega); \overline{\alpha}(\omega)] + j\cdot[\underline{\beta}(\omega); \overline{\beta}(\omega)]\\ &= [\underline{\alpha}_1; \overline{\alpha}_1] + [\underline{\alpha}_2; \overline{\alpha}_2] + [\underline{\alpha}_1; \overline{\alpha}_1]\cdot[\underline{\alpha}_2; \overline{\alpha}_2]\\ &- [\underline{\beta}_1; \overline{\beta}_1]\cdot[\underline{\beta}_2; \overline{\beta}_2] + j\cdot\{[\underline{\beta}_1; \overline{\beta}_1] + [\underline{\beta}_2; \overline{\beta}_2]\\ &+ [\underline{\alpha}_1; \overline{\alpha}_1]\cdot[\underline{\beta}_2; \overline{\beta}_2] + [\underline{\beta}_1; \overline{\beta}_1]\cdot[\underline{\alpha}_2; \overline{\alpha}_2]\ \}. \end{aligned}$$

Thus, the above result can be easily proved from the multiplication rule of interval sets. □

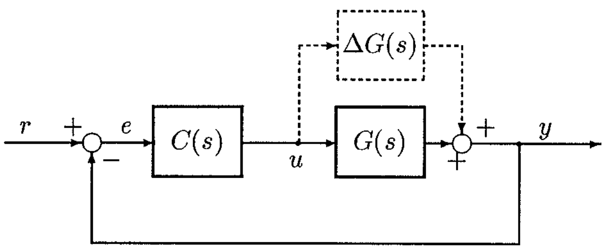

Fig. 4 Feedback control system with uncertain plant.

5. STABILITY ROBUSTNESS

Based on the foregoing assumption for the open loop frequency response characteristics with uncertainty, the stability of feedback control systems should be investigated. Consider a control system with $C(s)$ as an appropriate compensator as shown in Fig. 4. The closed loop characteristic equation becomes

$$1 + G(s)C(s)(1 + \Delta(s)) = 0. \qquad (22)$$

Concerning the complementary sensitivity function

$$T(s) = \frac{G(s)C(s)}{1 + G(s)C(s)},$$

it is well known that robust stability condition is written as

$$\|\Delta(s)T(s)\|_\infty < 1, \qquad (23)$$

where $\|\ \cdot\ \|_\infty$ denotes a H_∞ norm, that is,

$$\|f(s)\|_\infty := \sup_{\sigma>0}\cdot\sup_{\omega}|f(\sigma + j\omega)|. \qquad (24)$$

For a stable nominal system, the robust stability condition concerning radius $r(\omega)$ of the uncertain term easily becomes

$$|\Delta(j\omega)| = r(\omega) \le \overline{r}(\omega),\quad |T(j\omega)| < \frac{1}{\overline{r}(\omega)}. \qquad (25)$$

As for the modified system described in section 3, it can be written as

$$\left|\frac{G_m(j\omega)C(j\omega)}{1 + G_m(j\omega)C(j\omega)}\right| < \frac{1}{\overline{r}_m(\omega)}, . \qquad (26)$$

where

$$|\Delta_m(j\omega)| = \left|\frac{\Delta_c(j\omega)}{G_c(j\omega)}\right| \le \overline{r}_m(\omega).$$

[Theorem 4] When considering the Nyquist plot of $G_m(j\omega)C(j\omega)$, that is,

$$G_m(j\omega)C(j\omega) = U_m(\omega) + jV_m(\omega), \qquad (27)$$

the robust stability conditions, Eqs. (23) and (26) are rewritten as follows:

$$|\Delta(j\omega)G_m(j\omega)C(j\omega)| \le \rho_m(\omega) < |1 + U_m(\omega) + jV_m(\omega)|, \qquad (28)$$

where $\rho_m(\omega) = \overline{r}_m(\omega)|G_m(j\omega)C(j\omega)|$.
(Proof) Eq. (28) is obvious from Eqs. (26) and (27). □

On the other hand, using Kharitonov's concept the stability robustness can be evaluated as follows. When the uncertain terms in Theorem 4 are written as interval sets;

$$\begin{aligned} \Delta_m(j\omega)G_m(j\omega)C(j\omega) &= \alpha_{mc}(\omega) + j\beta_{mc}(\omega),\\ \alpha_{mc}(\omega) \in [\underline{\alpha}_{mc}; \overline{\alpha}_{mc}],&\quad \beta_{mc}(\omega) \in [\underline{\beta}_{mc}; \overline{\beta}_{mc}], \end{aligned}$$

the robust stability condition can be also represented by the following theorem based on the Nyquist stability criterion.

[Theorem 5] When the following inequality holds for the Nyquist locus:

$$\max_{\omega\in[\omega_1;\omega_2]} \overline{\alpha}_{mc}(\omega) < |1+U_m(\omega)| \quad (29)$$

for all $\omega \in [\omega_1;\omega_2]$ satisfying

$$V_m(\omega)+[-\overline{\beta}_{mc}(\omega);\overline{\beta}_{mc}(\omega)]=0, \quad (30)$$

the feedback control system is guaranteed to be stable.
(Proof) This result can be obtained from the Nyquist criterion, and also the Kharitonov theorem [5] [6]. □

6. NUMERICAL EXAMPLES

[Example 1] Consider the following controlled systems with first-order delay:

$$\begin{aligned} G_1^*(s) &= \frac{5}{(1+5s)(1+\tau_1 s)}, & (31)\\ \tau_1 &\in [\underline{\tau}_1;\ \overline{\tau}_1]=[0.0;\ 0.4], \\ G_2^*(s) &= \frac{8}{(1+10s)(1+\tau_2 s)}, & (32)\\ \tau_2 &\in [\underline{\tau}_2;\ \overline{\tau}_2]=[0.0;\ 0.3]. \end{aligned}$$

In this example, it is assumed that only the time constants τ_1, τ_2 are uncertain and represented by interval sets.

Obviously, the frequency response characteristics of uncertain terms Δ_1 and Δ_2 can be written repectively as follows:

$$\Delta_1(j\omega)=\frac{-j\tau_1\omega}{1+j\tau_1\omega},\quad \Delta_2(j\omega)=\frac{-j\tau_2\omega}{1+j\tau_2\omega}.$$

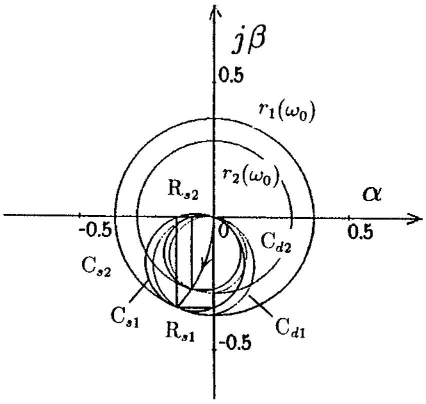

Fig. 5 Rectagular, circular and sectorial areas for uncertain term $\Delta_1(j\omega_0)$ and $\Delta_2(j\omega_0)$.

As for a fixed $\omega=\omega_0$; $\omega_0=1.0$ rectangles R_{s1}, R_{s2}, circles C_{s1}, C_{s2} and circles C_{d1}, C_{d2} are as shown in Fig. 5.

When modifying the nominal system as described in section 3, the following can be given:

$$G_1^*=G_{m1}(1+\Delta_{m1}),\quad G_2^*=G_{m2}(1+\Delta_{m2}).$$

Here, the modified frequency response characteristics G_{m1}, G_{m2} are written as

$$G_{m1}=\frac{5(1+0.2j\omega)}{(1+5j\omega)(1+0.4j\omega)},$$
$$G_{m2}=\frac{8(1+0.15j\omega)}{(1+10j\omega)(1+0.3j\omega)},$$

and the uncertain terms Δ_{m1}, Δ_{m2} can be expressed as follows:

$$\Delta_{m1}=\frac{-0.2j\omega}{1+0.2j\omega}=\alpha_{m1}+j\beta_{m1},$$
$$\overline{\alpha}_{m1}=\frac{0.04\omega}{1+0.04\omega^2},\quad \overline{\beta}_{m1}=\frac{0.2\omega}{1+0.04\omega^2},$$
$$\Delta_{m2}=\frac{-0.15j\omega}{1+0.15j\omega}=\alpha_{m2}+j\beta_{m2},$$
$$\overline{\alpha}_{m2}=\frac{0.0225\omega}{1+0.0225\omega^2},\quad \overline{\beta}_{m2}=\frac{0.15\omega}{1+0.0225\omega^2}.$$

When these systems are connected in cascade as shown in Fig. 3, the composited frequency response characteristics are expressed by rectangle and circle arrays as shown in Fig. 6. As for a fixed ω; $\omega_0=1.0$, the rectangular and the circular areas are as shown in Fig. 7 and Fig. 8.

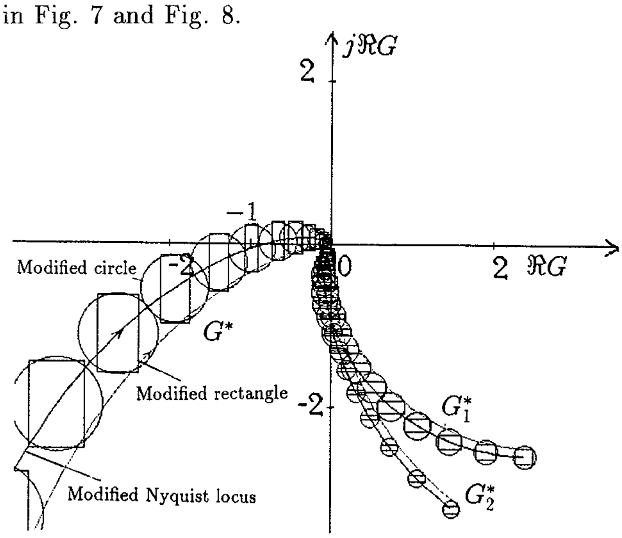

Fig. 6 Rectagle and circle arrays for a composited system in Example 1 (ω: 0.2 → 5.0).

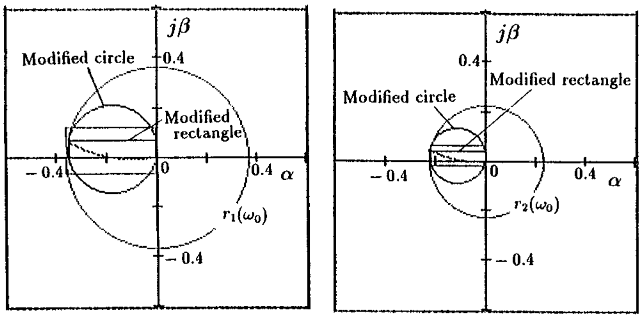

Fig. 7 Rectagular and circular areas for $\omega_0 = 1.0$.

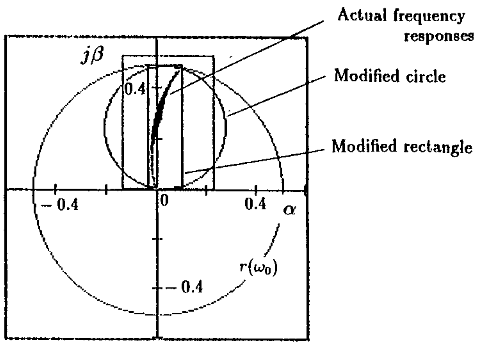

Fig. 8 Rectagular and circular areas of a composited system for $\omega_0 = 1.0$.

When the composited system described above is fed back by unity gain, i.e., $C(s) \equiv 1$, the stability of the closed loop system cannot be guaranteed from Theorem 4 and Theorem 5 as shown in Fig. 6.

[Example 2] Consider the case where the following controller (compemsator) is connected in cascade to the controlled system G_1^*:

$$C(s) = 0.5 \cdot \frac{1+2s}{1+s}. \tag{33}$$

In this case, rectagle and circle arrays for the compensated system is as shown in Fig. 9. As is obvious from the figure, the stability robustness is guaranteed.

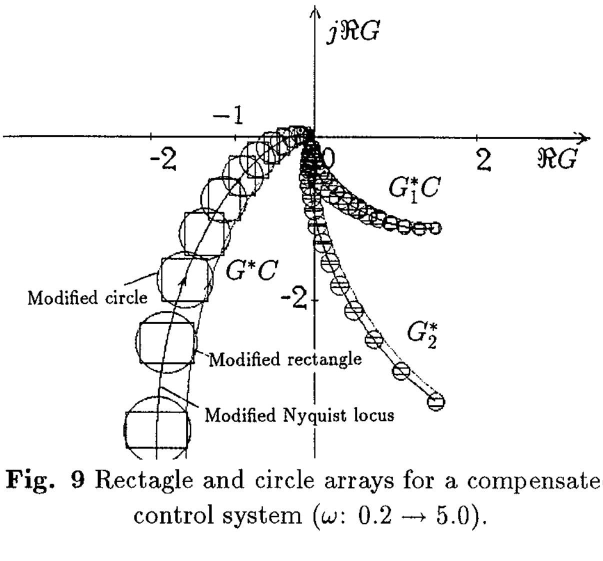

Fig. 9 Rectagle and circle arrays for a compensated control system (ω: $0.2 \to 5.0$).

7. CONCLUSIONS

This paper described the composition rule of frequency response characteristics represented by the interval sets based on the experimental data of controlled systems with uncertainty and nonlinearity. Then, the stability robustness of feedback control systems was graphically evaluated by using Kharitonov's concept. Not only the band of the gain characteristic but also the band of the phase characteristic is assumed to be given in the sets of the experimental data. Considering information on the phase enables to determine the robustness of the control system with uncertainty in a less conservative form.

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