Simple estimation of elastic parameters of soils and their effects on the immediate settlement of the soft ground

by

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Abstract

It is emphasized that immediate settlements or undrained deformation due to strip embankment on soft clay can occur even when Poisson's ratio ν is not 0.5. A method for evaluating elastic parameters using results from oedometer tests and unconfined compression tests is presented. Effects of Poisson's ratio on immediate settlements are examined on the basis of solutions by finite element analyses. Results are compared with those from previous studies in which it is assumed that $\nu=0.5$.

Key words : Elasticity, Immediate settlement, Soft ground, Oedometer test, Unconfinred compression test

1. Introduction

It seems reasonable to assume that the immediate settlement of saturated soft clay deposits occurs under the undrained condition because of the poor permeability of the soft clay.

The use of the theory of elasticity can be useful and convenient for preliminary designs such as the prediction of the immediate settlement of embankment on the soft clay layer.

Even if the theory of elasticity is used for such objects, elastic parameters have to be determined without disregarding the principle of effective stress. To do so, we have to determine them on the basis of relationships between effective stresses and strains. A proper method to the problem to be treated should be adopted for evaluating elastic parameters.

Since Skempton and Bjerrum (1957) proposed a method for predicting immediate settlements, the condition of undrained deformation is usually replaced by the condition that Poisson's ratio ν is equal to 0.5. However, the condition of undrained deformation is not inherent in soils but constraint given to soils. Accordingly we can not define values for constitutive parameters, for example, ν. In other words the immediate settlements can occur even if ν is not equal to 0.5.

It should be noted that we here use the theory of elasticity not because soils are considered to be elastic but because the simple form of the theory is preferred.

In the former part of this paper, a method for evaluating elastic parameters will be presented. In the method, results of practical laboratory tests such as oedometer tests and unconfined compression tests are used; and elastic constants relating effective stresses to strain are derived.

In the latter part of the paper, effects of the magnitude of ν on the immediate settlement will be discussed on the basis of results from finite element analyses applied for model problems. Furthermore, the results by a conventional method in which ν=0.5 is assumed will be compared with those from finite element analyses; and it will be shown that the conventional method can give practically satisfactory solutions in some cases although it can underestimate immediate settlements in other cases.

2. Evaluation of Elastic Parameters

2.1 Definitions

The coefficient of volume change m_v and undrained deformation modulus E_u are obtained in oedometer tests and undrained triaxial compression tests including unconfined compression tests. They are defined as:

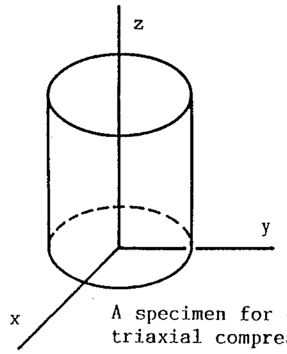

Fig.1 Coordinate system adopted.

$$m_v = \frac{\Delta\varepsilon}{\Delta p} \tag{1}$$

$$E_u = \frac{\Delta q}{\Delta\varepsilon} \tag{2}$$

where p is the consolidation pressure in oedometer tests and q is deviator stress in undrained triaxial compression tests; and ε is compressive strain in both tests. Undrained deformation modulus E_u is the tangent modulus in the initial portion of deviator stress and strain relations under undrained condition.

Isotropic and linear elasticity is assumed for the incremental relations between effective stresses and strains. For the convenience of description, we consider the cartesian coordinate axes x,y and z so that the axis z coincides with the vertical direction and x and y are in the horizontal plane (see Fig.1). In the oedometer tests and undrained triaxial tests, planes normal to these axes are principal ppanes of stress and strain.

For linear elastic materials the vertical component of stress increments is given as:

$$\Delta\sigma'_{zz} = (\lambda + 2\mu)\Delta\varepsilon_{zz} + \lambda(\Delta\varepsilon_{xx} + \Delta\varepsilon_{yy}) \tag{3}$$

where λ and μ are Lame's constants. An alternative form of stress strain relationships is derived using deviatoric components of stress and strain as follows:

$$\Delta\sigma_{zz} = 2\lambda\Delta\varepsilon_{zz} \tag{4}$$

2.2 Relationships of m_v and E_u to elastic constants

In the oedometer tests, the following conditions are held:

$$\Delta\varepsilon_{zz} = \Delta\varepsilon \quad (5.1)$$

$$\Delta\varepsilon_{xx} = \Delta\varepsilon_{yy} = 0 \quad (5.2)$$

$$\Delta\sigma_{zz} = \Delta p \quad (5.3)$$

By inserting Eqs.(5.1) to (5.3) into Eq.(3), we can express m_v in terms of elastic constants as:

$$m_v = \frac{1}{\lambda + 2\mu} \quad (6)$$

The definition for m_v, Eq.(1), was used.

For undrained triaxial compression tests with constant cell pressure, and, in a special case, for unconfined compression tests, we have the following conditions:

$$\Delta\sigma_{zz} = \Delta q \quad (7.1)$$

$$\Delta\sigma_{xx} = \Delta\sigma_{yy} = 0 \quad (7.2)$$

$$\Delta\varepsilon_{zz} = \Delta\varepsilon \quad (7.3)$$

$$\Delta\varepsilon_v = 0 \quad (7.4)$$

where ε_v is volumetric strain. Inserting these conditions into Eq.(4), we have:

$$E_u = 3\mu \quad (8)$$

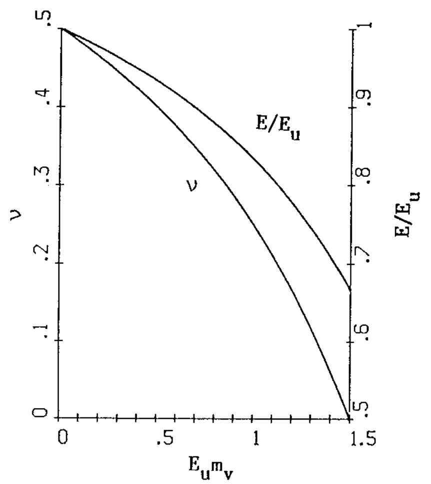

Fig.2: Variations of Poisson's ratio ν and normalyzed Young's modulus E/E_u with $E_u m_v$.

2.3 Results

It will be convenient to derive Young's modulus E and Poisson's ratio ν in terms of m_v and E_u. By expressing Lame's constants in terms of E and ν, we can derive following relations from Eq.(6) and (7):

$$\nu = \frac{2E_u m_v - 3}{2(E_u m_v - 3)} \quad (9)$$

$$E = E_u \frac{4E_u m_v - 9}{3(E_u m_v - 3)} \quad (10)$$

The following relation is also derived from Eqs.(9) and (10):

$$E = \frac{2}{3} E_u (1 + \nu) \quad (11)$$

Poisson's ratio ν and normalized Young's modulus E/E_u were calculated, according to Eqs.(9) and (10), as a function of the non-dimensional parameter $E_u m_v$. Results are presented in Fig.2. The range of the value for the parameter $E_u m_v$ was determined by the condition that $0<\nu<0.5$. This figure shows that both ν and E/E_u decreases with the increase in $E_u m_v$.

From this figure we can evaluate the values for ν and E using values obtained from oedometer tests and unconfined compression or undrained triaxial compression tests.

Eq.(11) means that, when $\nu = 0.5$, the Young's modulus E becomes equal to E_u. This is also confirmed in Fig.2.

3. Effects of ν on the Immediate Settlement

As was mentionedin the Introduction, immediate settlements can occur even when Poisson's ratio ν is not equal to 0.5. Effects of ν on the immediate settlement will be examined by using results from finite element elastic analyses in which small strain was assumed.

3.1 Method for analysis

The value of ν was varied from 0.1 to 0.5; In the calculations the condition of $\nu=0.5$ was replaced by that

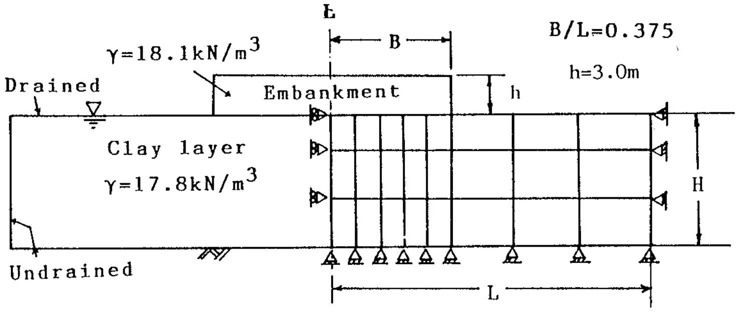

Fig.3: Domain for analyses and finite element mesh.

of ν=0.4999. The value of E was also varied but the results obtained under the condition of E=1.47 MN/m^2 will be shown.

The problem of a strip loading by embankment on a soft, saturated and homogeneous ground was solved. The domain and finite elements used in analyses are shown in Fig.3. Half of the breadth of the embankment B and the depth of soft subsoil H were varied; the ratio B/L was held constant. The total number of elements was not varied.

The equilibrium equations for stresses and continuity equation for pore water were discretized with finite element technique, unknown functions being displacement and pore water pressure. The displacement was approximated with a quadratic interpolation function and the pore pressure with linear one in an element.

For the current purpose, the continuity condition of pore water was replaced by the condition that volume change does not occur in elements.

3.2 Calculated results

The immediate settlement will be represented by that at the node just beneath the center of embankment and denoted by ρ_i.

In Fig.4, ρ_i/H is plotted against B/H. For a particular value of ν, the difference in the value of B or H does not appear in this diagram. Hence ρ_i/H is a function of E, ν and B/H.

We can see in Fig.4 that the relationships between ρ_i/H and B/H strongly depends on ν . For a particular value of ν, ρ_i/H varies with B/H, which represents the geometrical condition, in a complicated manner.

In the relatively low range of B/H such as B/H<1, ρ_i/H tends to increase with the increase in ν. On the other hand, ρ_i/H decreases with increasing B/H for B/H up to 3. For B/H larger than 3, the tendency is quite different between the case of ν=0.5 and other cases of ν<0.5.

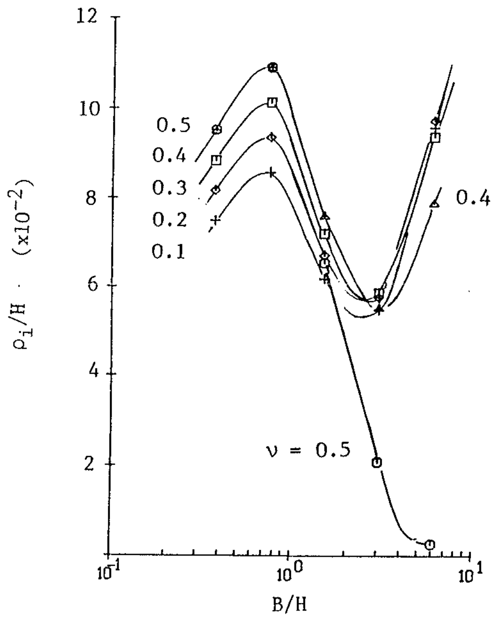

Fig.4: Relationships between ρ_i/H and B/H.

When so high values as B/H>3 are encountered, problems will be solved as one-dimensional ones. Thus we are practically interested in the geometrical condition of B/H<3.

In order to visualyze the dependence of ρ_i/H upon ν, Fig.5 was prepared, in which the ratio of ρ_i/H to ρ_i/H(ν=0.5) is plotted against B/H. In this figure the range for B/H larger than 3 is not shown. We can see that immediate settlement tends to decreases with decreasing in n for any particular geometrical condition.

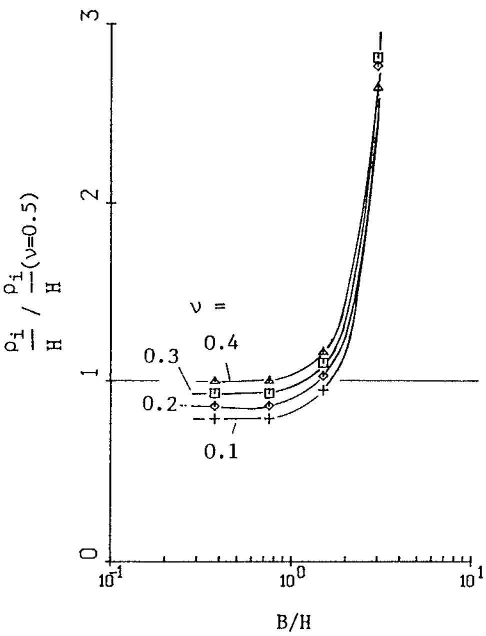

Fig.5: Effects of ν on the immediate settlements.

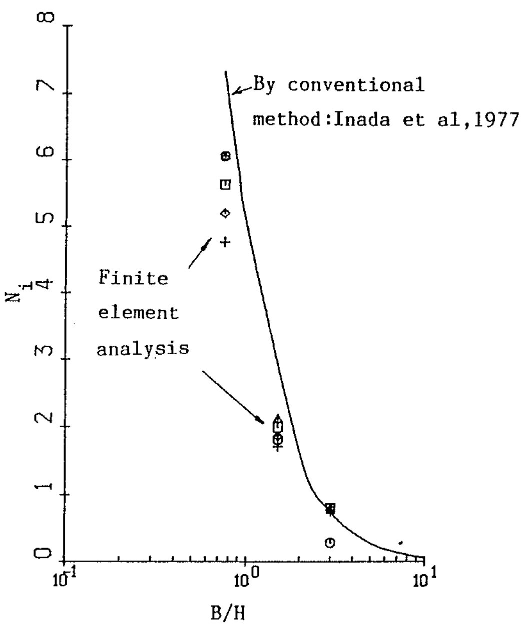

Fig.6: Comparison between numerical solutions and an analytical soulution.

4. Discussion

If the value of E_u is determined for infinitely small strain, according to the definition of Eq.(2), it can correctly correspond to Young's modulus. In practice, however, it is not easy to determine E_u for such small strain. In addition to this, our purpose is to crudely estimate elastic parameters. Thus we may use E_{50} in place of E_u, where E_{50} is secant modulus corresponding to half of the maximum of q.

This is also justified because relatively large strains can occur in cases such as instantaneous embankments on a soft ground. Inelastic components will be possibly included in large strains.

It is simple to estimate elastic parameters from Fig.2. However, according to some experiences by the Authors, some data can lead to the condition of $E_{50}m_v>1.5$. For such a case we can not use Fig.2. Various reasons could be cited for this; one major is probably due to some disturbance that samples used had subjected to.

In almost all studies made for the prediction of immediate settlement of soft ground, solutions to the problem of elastic half space of infinitely large depth and breadth are adopted.

Davis and Poulos (1968) considered the layered subsoils; they pointed out that the total compression of a layered subsoil should be evaluated by summing up the strain in each layer over the finite depth of the subsoil.

Inada et al. (1977) proposed an interesting method in which the finiteness of the depth of subsoil is taken into account: they first derived principal strains from stress solutions obtained for the infinitely elastic half space and integrated them over the finite depth of the subsoil considered.

They expressed immediate settlement in the form of:

$$\rho_i = \frac{q}{E} N_i \qquad (12)$$

where q is the applied surface load and N_i is the coefficient of immediate settlement having thedimension of the length.

In Fig.6, where N_i is plotted against B/H, numerical solutions obtained in this study are compared with analytical solutions by Inada et al.(1977). The N_i-B/H diagram depends on the magnitude of B and a representative case of B=15 m is shown.

Analytical solutions tends to overestimate the immediate settlement in the examined range of B/H. However the deviation from the numerical solutions are rather small. For values of B other than 15 m, the deviation can be larger.

In the above we made an attempt to compare the numerical solutions with analytical ones. It should be noted here that the direct comparison between them can be unreasonable because the domain treated in numerical analyses is not infinitely large. In addition to this, finite element solutions always possesses inherent errors associated with the discretization.

5. Conclusions

A method for simply estimating elastic parameters was presented. It seems that the method can be applied to preliminary design problems, for instance, immediate settlements due to embankment on soft caly subsoill.

It was emphasized that undrained deformation can occur in subsoils even if Poisson's ratio ν is not equal to 0.5. Effects of ν upon the immediate settlement was discussed by using solutions from finite element analyses.

When the breadth of embankment B is less than about two times the depth of subsoil D, the immediate settlement decreases with decreasing ν. For higher ratio B/D, the dependence on ν is complicated.

The numerical solutions from finite element analyses were compared with an analytical solution that was obtained by assuming ν=0.4999 for undrained deformation. Analytical solution gave practically satisfactory results.

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