Applying Neural Network Modeling to Route Navigation

by

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Neural networks, neurocomputing is based on the wistful hope that we can reproduce at least some of the flexibility and power of the human brain by artificial means. Recently much interest has been focused on the development of advanced traveler information systems using neural network technique. This paper presents a neural network model to predict traveling time on networks and to investigate the impacts of route guidance systems upon drivers' long-term behavior. The model explicitly takes into account drivers' learning behavior. The prototype model of route navigation information systems are designed through simulation experiments. The paper concludes by assessing pros and cons of neural network modelling to route navigation systems and by suggesting further research subjects.

Key words : Neural network, Route guidance system, Time series, Rational expectations, Bayesian rules

1. Introduction

Transport networks are increasingly faced with the problems of congestion externalities which tend to reduce the overall performance of networks. Thus, the advanced traveler information system (ATIS) have been investigated in many countries as a high-tech approach to aid drivers more informative route choices and to alleviate increasing levels of traffic congestion on networks. This study has tried to design a prototype model for route navigation systems. In this paper, our focus are upon the development of prediction system of traveling time of routes by using neural network technique. The neural network systems are constructed through a series of computer experiments, which hypothetically simulate the fluctuation of network flow and realize individual drivers route choices. In designing simulation experiments, only one O-D pair connected by two routes with different characteristics are considered. The overall turbulence of travel time by local traffic is assumed to obey a certain type of stochastic process and is generated by a time series model. Through experiments, the optimum configuration of the neural network architecture is determined. A three-layer, feed forward neural network with a back-propagation learning algorithm is coded in the C language on P.C. The three-layer neural network was implemented with 4 neurons in the input layer and two neurons in output layer, corresponding to the number of attributes in the input vector and the number of output alternatives, respectively. The number of neurons used in the experiments ranged from 1 to 4 for the first layer and from 1 to 4 for second hidden layer. This paper also tries to provide with a unified framework for understanding how drivers act in response to exogenously provided route guidance information; and how they form subjective expectations on traffic conditions from repeated learning. The paper is organized as follows. In Section 2, the basic analytical strands for designing route navigation systems are described. Section 3 formulates drivers' route choice behavior and Section 4 explains how the paper models the drivers' learning processes in the simulation experiments. In Section 5 the neural network model is presented by which the public agency can provide drivers with route guidance information. In Section 6, the results of simulation experiments are summarized.

2. Analytical Strand

2.1 Information and Route Choices

An important feature of existing traffic assignment models is the representation of the interactions between link travel cost and link traffic volume. Most of these models have been concerned with predicting average conditions over a period of time rather than actual conditions on a particular day. Many of them have sought to generate equilibrium flow patterns which might be expected to come about after a period of time. Recent developments in assignment modeling have been more concerned with the incorporation of network dynamics and stochastic choices. The stochastic user equilibrium (SUE) approach firstly formulated by Daganzo and Sheffi (1977)[1] has been extended to cope with dynamic network modeling by incorporating driver behavior models and network performance models. This framework explicitly treats the distribution of traffic by time-of-day and the drivers' pretrip and en-route adjustment process (e.g., Fisk 1980[2]; Sheffi 1985[3]; Ben-Akiva et al. 1991[4]).

Given what has been learned from the models and empirical works, it is now the time to develop a more comprehensive framework for understanding drivers' route choice behavior with and without route guidance information. The basic form of the model is determined by the need to be able to represent the performance of route guidance information systems in the context of sporadic and dynamically evolving congestion. It follows that the models must represent drivers' perceptions and expectations as they might evolve on a particular day rather than being concerned with average or equilibrium conditions. If the average performance of the system over a period of time is required it will be necessary to consider a number of days and then derive an average performance rather than take an average day. There has been no obvious analytical solution to this and a dynamic simulation of drivers' learning and fluctuations in their route choices over periods therefore seems necessary.

The basic rationale behind this belief is that many drivers possess little or no reliable information concerning travel routes and alternative travel decisions. As drivers' decisions are affected by expected network conditions, the most useful type of information to a driver faced with travel choices would be reliable predictive information. Predictive information must be based on projected traffic conditions which are dependent on the ways in which drivers will respond to the information. The validity of predicted information depends on their consistency with current and future drivers' choices which depend on their use of such information. Thus, the relationships among drivers expectations, information reliability and drivers' behavior need to be models in a way which explicitly describes drivers' evolving perceptions and learning mechanisms.

2.2 Problem Setting

Route guidance information systems have the potential of reducing or eliminating poor route choices and consequently excess travel distance and cost incurred by unaware or uninformed drivers. However, it is very likely to happen the concentration of traffic on the recommended routes and the overreaction of drivers in their response to guidance information. It is also expected that the reliability of the guidance system will be faded away as the fraction of informed drivers increases. Hence, the impacts of the guidance information itself on drivers' perceptions and expectations need to be explicitly taken into consideration if one tries to design navigation systems providing drivers with route guidance information.

Consider a situation in which a small number of drivers start to receive route guidance information. Assume that the information provided is unbiased. The information corresponds to various signals which reduce or eliminate uncertainty. When a driver is able to efficiently use this information he or she is better off. However a driver may be unable to process this information to select the optimal route if he or she may be distracted by the large amount of available information. Thus, information need to be provided drivers with in an understandable (stylized) way. Consider a more complicated situation in which the majority of drivers receive public information on traffic conditions. In this case, it is very likely to happen that drivers overcorrect their beliefs and drivers' overreaction to public information may cause congestion to transfer from one road to another. Overreaction happens if too many drivers respond to public information on current traffic conditions. It may also generate oscillations in road usage.

The above descriptions outline a number of important questions which must be addressed in relation to future developments of electronic route guidance systems. Is it possible to provide drivers with reliable predictive information? Do we have the tools to provide predictive information which is consistent with realized traffic conditions? If the above two questions are answered affirmatively, then when, how frequently, and to whom should such information be provided? The fraction of drivers which should be informed is an important policy variable.

More predictive information is most costly, but may decrease the possibility of an overreaction. When drivers with communication devices receive public information and alter their behavior, they affect driving conditions for others, both those with devices and those without. Moreover, if uninformed drivers know that informed drivers are out there they may adjust their behavior too, albeit on a routine rather than daily basis because they lack day-specific information. This may cause informed drivers to make further adjustments, and so on. Thus, the reliability of the route guidance information systems are endogenously determined by the whole drivers' behavior within urban networks. A driver's behavior changes over time, often from day-to-day, due to learning, expectations formation, variable perception of the reliability of the information received, etc. Thus, in broad terms, any framework aimed at analyzing the potential impacts of route guidance information systems should incorporate dynamic models of drivers' behavior and expectations formation.

2.3 Type of Information

When making route choices, drivers constantly combine sources of information to form perceptions and expectations of traffic conditions. Conventional sources of information available to drivers include personal experience, word of mouth, and media messages. Drivers who rely solely on such information are likely to have incomplete information about traffic conditions on the network. Information available to drivers may conceptually fall into three categories: (1) historical information - information describing the state of the transportation system during previous time periods; (2) current information - the most up-to-date information about current traffic conditions; (3) predictive information - information concerning during subsequent time periods when travel can occur. Another classification is also useful for our current purpose: a dichotomy of information into that category of common (public) information and private information. Public information, like knowledge about networks, is, in principle, available to the public, and forms a part of the common knowledge for all drivers in Aumann's sense (Aumann, 1976)[5]. That category of private information may include a broad spectrum of a driver's information totally hidden to others. A driver's preference, characteristic, historical information and prediction may be classified into this category.

If a state of nature is known and the driver's choices of routes are known to all, each driver will know the travel cost of the available routes and his corresponding utility. In contrast to the single driver decision problem, we consider multi drivers. Thus, a complete description of a state of nature must contain information not only for resolving the uncertainties, but also for determining the extent to which each driver knows the state of nature. There must be some states of nature that are distinguishable from others if there is incomplete information. The degree to which the natural states are indistinguishable will affect the drivers' behavior and must be part of the description of a natural state. Recognizing the possible ability of drivers to differentiate among states allows us to analyze asymmetric information beyond that treated by standard decision theory. A complete description of a network should resolve these uncertainties.

In a world not subject to incomplete information, drivers need look no further than their own preferences to be able to make a decision. They need give no thought to the actions of other drivers. However, in a world subject to incomplete information and random fluctuations, this is no longer the case. Drivers are faced with the problem of forecasting travel conditions which are dependent on the actions of other drivers. Rational expectations theories provide for a model of how drivers make these forecasts. Furthermore, in a world of incomplete information, drivers possibly try to acquire information about the future realization of travel conditions. It will, in general, be the case that different drivers have access to different information. The fact that information is dispersed throughout the drivers has the potential to cause a misallocation of route choices relative to what would be the case if all drivers know everything. An efficient allocation of route choices will in general require the transfer of information from the public agent who has some information about the fluctuations of the traffic conditions to individual drivers who can take current actions to mitigate avoidable congestions.

2.4 Rational Expectations Hypothesis

The past decade has witnessed important developments in the study of the expectations formation processes and the problem of decision-making under uncertainty. Of the theories of expectations formation so far advanced, the rational expectations hypothesis has attracted by far the greatest attention. The rational expectations hypothesis (REH) due to Muth (1961)[6] states that subjective expectations held by economic agents will be the same as conditional mathematical expectations based on the true probability model of the economy; or more generally - that the agents' subjective probability distribution coincides with the objective probability distribution of events. Although the REH was advanced by Muth it was work of the Lucas (1978)[7], Sargent (1973)[8], Barro (1976)[9] and others that brought it into prominence. This paper makes no attempts to survey the literature on rational expectations. The reader is referred to Shiller (1978)[10] for a macroeconomics and Sheffrin (1983)[11], Radner (1980)[12] for a survey of the microeconomics.

In the given context of drivers' route choice behavior, REH assumes that a driver who has a good understanding of a network can efficiently utilize his daily experience to make inferences about the consequence of the route choices taken by other drivers. These inferences are derived, explicitly or implicitly, from an individual's model of the relationship between the information received by himself and the traffic conditions realized in the network. On the other hand, the true relationship is determined by the individual drivers' behavior, and hence by their expectations. The drivers have the opportunities to revise their expectations in the light of observations. Hence, there are feedback routes from the true relationship to the individual expectations. An equilibrium of this system, in which the individual expectations are identical to the true distributions, is called a rational expectations equilibrium (REE). In what follows, we characterize a network equilibrium with incomplete information where the respective drivers may form the rational expectations about traffic conditions.

3. Rational Expectation Equilibria

3.1 Information Structure

In this section, we presents a new analytical framework for network equilibria with rational expectations. The basic element of our network equilibrium concept is differential information; different users have different information about the route traffic conditions; they choose their route on the basis of their private (differentiated) information. The purpose of this section is to develop a general equilibrium concept that makes explicit the information or knowledge that a user has as part of his primitive characteristics. The model we describe in this section is a reinterpretation of Harsany's model of incomplete information game (Harsanyi 1967-1968)[13]. The difference from Harsany's approach is the explicit consideration of the rational expectation formation by drivers (Kobayashi 1990)[14].

Consider N driver and the set of drivers S. Let us explain how one can formally describe a driver's information about other drivers' characteristics, preferences and route choices. Driver $s \in S$ has his/her own private information, $\omega_s \in \Omega_s$, which is not observable by others including the public agent. Let Ω be the set of all possible ω. For driver s, let $\Phi_s(\omega) : \omega \to \omega_s$ be an onto mapping defined on Ω. Let ω_s be the signal observed by driver s if ω occurs. Driver s can distinguish between ω' and ω'' if $\Phi_s(\omega') \neq \Phi_s(\omega'')$. If $\Phi_s(\omega) \neq \omega$, the private information space of driver s is called incomplete. Let us define the whole space of private information Θ which is defined by a product of drivers' private information spaces:

$$\Theta = \prod_{s=1}^{N} \Phi_s(\Omega) \tag{1}$$

Let us define information structure $\mu \in \Theta$ which is an explicit representation of the incompleteness of all drivers' private information spaces. The realization of driver s's private information and the information structure is represented by $\hat{\omega}_s (= \Phi_s(\hat{\omega}))$ and $\hat{\mu} = \prod_{s=1}^{N} \Phi_s(\hat{\omega}) \in \Theta$, respectively. Further, we assume that there are some common measures concerning the distribution of information.

3.2 Route Choice Behavior

Travel cost of each route is varying from time-to-time depending upon the fluctuations of local traffic volume and of individual choices. Travel cost of route a, $\tau_a(a \in \delta_s)$, is a random variable, and each driver is assumed to forecast the probabilistic distribution of τ_a. Driver s's subjective expectations on τ_a, $\pi_{as}(\psi)$, can be formalized in the form of probability density functions $\pi_{as}(\tau_a;\psi)$. The symbol ϕ designate the basic case where no route guidance information is provided. Given $\pi_{as}(\tau_a;\phi)$, the expected utility of driver s for route a is defined by

$$V(\hat{\omega}_{as};\phi) = \int U(\tau_a,\hat{\omega}_{as})\pi_{as}(\tau_a;\phi)d\tau_a, \tag{2}$$

where ω_{as} is driver s's private information on route a and U is a Neumann-Morgenstern type of utility function. Assume that $\partial U/\partial\tau_a \leq 0$ and $\partial^2 U/\partial\tau_a \geq 0$. Driver s is assumed to choose the route which maximizes his/her expected utility function (2).

Let us next investigate the drivers' behavior when route guidance information is provided to the public. Denote the information (message) transmitted to the drivers at each period by $e \in \eta$, where η is the set of messages. If the messages, for example, 'Choose rote 1 ($e = 1$)' and 'Choose route 2 ($e = 2$)' are concerned, then the set of messages is denoted by $\eta = (1,2)$. Denote also the subjective expectations for all available routes conditional on message $e \in \eta$ by a tuple of mutually independent probability density functions, $\pi_s(e) = \{\pi_{as}(\tau_a;e), a \in \delta_s\}$, and describe the whole spectrum of subjective expectations conditional on the message set η by $\Pi_s(\eta) = \{\pi_s(e); e \in \eta\}$. Each density function specifies a driver's subjective belief regarding a conditional distribution of travel cost given a message.

Consider a situation where the public agent inform drivers message $\hat{e} \in \eta$. Then, given the subjective expectations $\pi_{as}(\hat{e})$ in $\Pi_s(\eta)$, the expected utility of driver s for route a, $V(\hat{\omega}_{as};\pi_{as}(\hat{e}))$, can be represented by

$$V(\hat{\omega}_{as};\pi_{as}(\hat{e})) = \int U(\tau_a,\hat{\omega}_{as})\pi_{as}(\tau_a;\hat{e})d\tau_a. \tag{3}$$

The driver chooses the route which maximizes his/her expected utility (3). Then the route chosen by driver s is given by

$$\gamma^*_{as}(\hat{\omega}_s;\pi_s(\hat{e})) = \arg\max_a\{V(\hat{\omega}_{as};\pi_{as}(\hat{e}))\}, \tag{4}$$

where the symbol *arg* designates the route which can maximize the R.H.S. of (4). Extend the above discussions for a single driver to all drivers on the network. The Nash equilibria induced by the situation where all noncooperative drivers compete with each other with incomplete information on a network environment fully characterizes our equilibrium concept with incomplete information. Given the information structure $\hat{\mu}$ and the message $\hat{e}$, the set of the route chosen by all drivers - a network equilibrium with incomplete information - can be described by $\gamma^*(\hat{\mu};\pi(\hat{e})) = \{\gamma^*_s(\hat{\omega}_s;\pi_s(\hat{e}))\}_{s\in\hat{S}}$. Since, as have repeatedly explained, the information structure $\hat{\mu}$ is constituted by a set of private information, no one can have access ex ante to the whole results of route choice $\gamma^*(\hat{\mu};\pi(\hat{e}))$ in each period.

3.3 Rational Expectations and Equilibrium

After each choice, each driver is able to record not only his private information and public information, but also the realization of travel cost of each run. After m route choices of route a, the s-th driver obtains an m-size empirical sample from the objective distribution of travel cost of this route. Based on the empirical samples, driver s forms his/her subjective expectations on travel cost conditional on public information. A rational driver, sooner or later, will be motivated to revise his/her $\pi_{as}(\tau_a;e)$ if he notices that it differs from the objective distributions of travel cost conditioned on message e, $v_a(\tau_a;e)$, through learning process. If both all rational drivers' conditional expectations $\pi_{as}(\tau_a; e)$ and the conditional objective distributions $v_a(\tau_a; e)$ are simultaneously converge to the rational expectations $\pi_a^*(\tau_a; e)$, let us call that the system reaches to the rational expectations equilibrium conditioned on public information.

A formal characterization of network equilibrium with rational expectations appears in Kobayashi (1990, 1993)[14][15]. The existence of the rational expectation is guaranteed under fairly weak network conditions (Mertens et al. 1985[16]; Kobayashi 1993,1994[17][18][19]). Rational expectation is a condition of network equilibrium rather than being only a condition of individual rationality. In a rational equilibrium, the information requirements are no greater; drivers need only know the stochastic process generating travel cost. Though the theory of rational expectations equilibrium tells the public agent about the underlying structural factors that determine the distribution of travel cost, in equilibrium drivers need not know anything about the structural form of the system. They need only know the relationships between public information and stochastic factors that may determine network performances.

It must be noted here that the time interval between dates is short relative to the speed of adjustment of expectations, before expectations can adjust to a temporary network flow the system will already be at the next date, and the environment will have changed. One will then observe a process of repeated incomplete adjustment, together with stochastic changes in the environment, and the system will always be in disequilibrium in the sense that networks will never equilibrate. Nevertheless, even in this case of repeated disequilibrium one would want to distinguish situations in which travel cost and drivers' route choices fluctuated in some 'steady' manner around long-run averages, from situations in which travel costs or route choices, or both, fluctuated with greater and greater variance, or increased without bound. To describe the situation it is natural to use the concept of a stationary stochastic process, which is the generalization to the case of uncertainty of the concept of a deterministic equilibrium. However, it is important to emphasize that the stationarity of a stochastic process does not rule out fluctuations of varying period and amplitude. The drivers can learn what is happening around him/her and to form the rational expectations, since their decision environment is subject to a stationary stochastic process.

4. Rational Expectations Formation

4.1 Expectations Formation by Learning

A useful way to model a driver's learning process is to imagine that at the beginning of period t, driver s has his/her own subjective expectations on travel cost of each route and receives message e from the public agent. Suppose, at the period, the driver makes his/her choice γ_s^t based on his/her subjective expectations conditioned on message e. At the end of this period, he/she eventually observes travel cost $\tau_{\gamma_s}^t$. $\tau_{\gamma_s}^t$ is commonly observed by all drivers having chosen it, but not by other drivers. The drivers may update their subjective expectations, as far as they are motivated to revise it. But, the learning problem is a little bit complicated by the presence of unobserved routes.

We assume that the learning actions described in the above are repeated over periods, and in each sample period t ($t = 0, 1, 2, \cdots$) the adjustment process takes place, given all drivers' route choices. For adjustment stage t, driver s must use the data τ_{as}^t to form, in sample period $t+1$, new estimates of the conditional distributions of travel cost on public message e. A rule for estimating the conditional distribution is called an estimation procedure, and the entire array of estimation procedures for all drivers and all adjustment stages is an estimation scheme.

Let us show that in a given stationary environment, each driver's subjective expectations converge to the rational expectations through learning processes. Define here the set of historical information. Historical information designate the one which a driver can obtain by past experience. It comprises four types of information: (1) private information ω_s^t, (2) a route choice in period t, γ_s^t, (3) travel cost of chosen route $\tau_{\gamma_s}^t$, and (4) message provided by the public agent in period t, e^t. Designate the set of historical data which driver s obtains in period t through his/her choice by a triple of $\sigma_s^t = (\gamma_s^t, \tau_{\gamma_s}^t, \omega_s^t, e^t)$. Let $\Xi_s^t = \prod_{z=1}^{t-1}\{\sigma_s^z\}$ be the whole spectrum of historical information which driver s have compiled up to period t.

At each stage of the learning processes, the drivers' expectations to message e are determined by their estimation procedures and their historical information (experience) in previous sampling periods. For each e, driver s's initial beliefs are assumed to be $\pi_s^0(e) = \{\pi_{as}^0(\tau_a; e)\}_{a\in\delta_s}$. For each $t > 0$ the drivers' subjective expectations for message e is fully regulated by their past experience Ξ_s^t and an initial belief $\pi_s^0(e)$:

$$\pi_s^t(\tau; e) = \phi_s^t(\tau, e; \Xi_s^t, \pi_s^0), \tag{5}$$

where ϕ_s^t represents a 'expectations formation mechanism', which explains how driver s form his/her subjective expectations from his/her past experience and initial expectations. The recursive nature of the learning process is critical to the expectations formation. At each stage, learning affects the subjective beliefs being learned at the next stage, but there is no feedback from later to earlier stages. With a certain learning rule Υ, the expectations formation mechanism $\phi_s^t(\tau, e; \Xi_s^t, \pi_s^0)$ can be expanded in a recursive form:

$$\phi_s^t(\tau, e; \Xi_s^t, \pi_s^0) = \Upsilon\{\sigma_s^{t-1}, \Upsilon\{\sigma_s^{t-2}, \cdots, \{\Upsilon\{\sigma_s^1, \pi_s^0\}\}\cdots\}. \tag{6}$$

An estimation scheme is successful (a.s.) if, for almost every infinite sample, the estimates converge to the conditional distributions mentioned in our initial description of the adjustment process given above.

The problem of learning rational expectations is greatly complicated by the dependence of the correct conditional distributions on drivers' beliefs. Indeed, if drivers modify their subjective expectations through learning procedures, their route choice behavior will change. Eventually, the objective distribution of travel cost will change time after time. Thus, the bilateral relationships exist between the conditional subjective expectations and the conditional objective distribution of travel cost.

4.2 Specification of the Information Structure

Consider a discrete network with a finite number of nodes and links. Denote the set of drivers by $S = \{1, \cdots, N\}$ and the set of admissible routes for driver $s \in S$ by δ_s. Suppose that driver s chooses route $a \in \delta_s$ with his/her subjective expectations $\pi_{as}(\tau_a; e)$ given private information ω_t and public message e. Let use specify the expected utility function of driver s for route a given private information ω_{as} and message e in the additively separable form with respect to private information:

$$V(\omega_{as}; \pi_{as}(e)) = \int U(\tau_a)\pi_{as}(\tau_a; e)d\tau_a + \omega_{as}, \tag{7}$$

where ω_{as} is a random variable representing private information concerning route a.

If route choice probabilities are mutually independent, the distribution of link traffic volume can be approximated by a multi-variate normal distribution function (Sheffi 1985)[3]. If linear link-performance functions are applied, travel costs are also subject to normal distribution functions. For the case of non-linear link-performance functions, travel costs are generally subject to certain skewed distributions. Normal distributions can be regarded as the second-order approximation of arbitrary probabilistic distributions. Assume the independency among private information, i.e., $E[\omega_{as}, \omega_{a's'}] = 0$ $(a \in \delta_s, a' \in \delta_{s'})$). This property implies that private information conveys no information about others' behavior. In this notion, private information designates any local, accidental and non-memorable factors affecting current drivers' route choices. Private information is varying from day-to-day and causes the fluctuations of drivers' route choices. Let us specify the representative driver's deterministic utility function by

$$U(\tau_a) = 1 - \exp\{\zeta(\tau_a - E_s[\tau_a])\} - E_s[\tau_a], \tag{8}$$

where $E_s[\tau_a]$ is the expected value of τ_a with respect to his subjective expectations $\pi_{as}(\tau_a; e)$ and ζ is the measure of the absolute risk aversion. Equation (8) is the first-order approximation of the utility function at $E_s[\tau_a]$. Let us take the Taylor expansion of driver s's conditional expected utility function on message e (see (7)) around $E_s[\tau_a]$. Then it is approximated in the form of the additive sum of means $\pi_{1as}(e)$ $(= E_s[\tau_a])$, variances $\pi_{2as}(e)$ and private information ω_{as}. It is shown that

$$V(\omega_{as}; \pi_{as}(e)) = -\pi_{1as}(e) - \frac{1}{2}\zeta^2\pi_{2as}(e) + \omega_{as}. \tag{9}$$

where $\pi_{1as}(e)$ and $\pi_{2as}(e)$ characterize driver s's subjective expectations conditional on message e. The route chosen by driver s given $\hat{e}$ and $\hat{\omega}_{as}$ with the subjective expectations $(\pi_{1as}(\hat{e}), \pi_{2as}(\hat{e}))$ is described by

$$\begin{aligned} \gamma_s^*(\hat{\omega}_s; \pi_s(\hat{e})) &= arg\max_a\{V(\hat{\omega}_{as} : \pi_{as}(\hat{e}))\} \\ &= arg\max_a\{-\pi_{1as}(\hat{e}) - \frac{1}{2}\zeta^2\pi_{2as}(\hat{e}) + \hat{\omega}_{as}\}. \end{aligned} \tag{10}$$

4.3 Bayesian Learning Rules

Describe the drivers' mechanism for expectations formation ϕ_s^s by use of a Bayesian estimation method. In a Bayesian framework probability is defined in terms of a degree of belief. The probability of an event is given by an individual's belief in how likely or unlikely the event is to occur. This belief may depend on quantitative and/or qualitative information, but it does not necessarily depend on the relative frequent of the event in a large number of future experience. Because this definition of probability is subjective, different individuals may assign different probabilities to the same events.

As drivers can obtain more sample information through their daily route choices, they are able to update their subjective expectations to reflect this additional information. Bayesian learning procedures can be described in the form of updating rules of $(\pi_{1as}^t(e), \pi_{2as}^t(e))$ based on the new σ_s^t. This can be achieved using some convenient recursive formulae. Suppose that $(\pi_{1as}^t(e), \pi_{2as}^t(e))$ is the posterior parameters calculated from observations up to period t, and assume driver s accepts message e in period t and chooses route a. For the simplicity of expression, let us omit subscripts a, s, e for the moment. Then, for additional observations given (a, τ_a), the forecasting model in period $t+1$ is shown that

$$\pi_1^{t+1} = \frac{\nu_0\mu_0 + n^t\bar{\tau}_t}{\nu_0 + n^t}, \tag{11}$$

$$\pi_2^{t+1} = \left\{\beta_0 + \bar{s}_t^2 + \frac{\nu_0 n^t}{\nu_0 + n^t}(\bar{\tau}_t - \mu_0)^2\right\}/\alpha_t, \tag{12}$$

where $\alpha_t = \alpha_0 + n^t/2$, $\bar{\tau}_t = 1/n \cdot \sum_{j=1}^{n^t}\tau_j$, $\bar{s}_t^2 = \sum_{j=1}^{n^t}(\tau_j - \bar{\tau}_t)^2$ and n^t is the number of observations on route a up to t-th period. Once these quantities have been obtained it is straightforward to derive the updating formulae. By expanding equations (11), the learning rule Υ of mean π_1^t can be given by the following recursive formula:

$$\pi_1^{t+1} = \pi_1^t + \frac{1}{\nu_0 + n^t} \cdot (\tau_t - \pi_1^t). \tag{13}$$

The subjective expectation π_1^t is revised by use of the forecasting error $(\tau_t - \pi_1^t)$ to yield the new expectation π_1^{t+1}. As shown by (13), the weight $1/(\nu_0 + n^t)$ is no more constant. As n^t becomes large, the significance of the forecasting error in revising the subjective expectations will decrease and eventually the weight converges to 0.

Analogous results for π_2^t can be derived from equation (12):

$$\pi_2^{t+1} = \pi_2^t + \frac{1}{\alpha_t}\{\frac{\nu_{t-1}}{\nu_t}(\pi_1^t - \tau_t)^2 - \frac{\pi_2^t}{2}\}, \tag{14}$$

where $\alpha_t = \alpha_0 + n^t/2, \nu_t = \nu_0 + n^t$. Note from equation (14) that the posterior variance π_2^{t+1} can be given by modifying π_2^t based on the forecasting error $\nu_{t-1}(\pi_1^t - \tau_t)^2/\nu_t - \pi_2^t/2$. As drivers accumulate historical information, the weight $1/\alpha_t$ will converge to 0. The definition of the recursive formulae (13) and (14) embodies the assumption that the driver having chosen route a never observe $\tau_j^t (j \neq a)$ even after τ_j^t is realized. Hence, the subjective expectations for route $j(\in \delta_s) \neq a$ are not updated until it will be chosen. That is, for $j(\in \delta_s) \neq a$ we assume that

$$\pi_{1js}^{t+1} = \pi_{1js}^t, \quad \pi_{2js}^{t+1} = \pi_{2js}^t. \tag{15}$$

If t becomes sufficiently large, from equations (11) and (12), we know that π_1^t, π_2^t can be approximated by

$$\pi_1^t \simeq \bar{\tau}_t, \quad \pi_2^t \simeq \frac{\bar{s}_t^2}{n^t}, \tag{16}$$

where $\bar{\tau}_t$ and $(\bar{s}_t^2/n^t)$ are the sample mean and variance, respectively. As drivers obtain more observations, their subjective means and variances asymptotically converge to the objective ones, respectively. Thus, given a triple of initial subjective expectations, the rational expectations appear as the limits of drivers' learning procedures.

5. Neural Network Model for Route Navigation

5.1 Information Systems for Route Navigation

In order to investigate the impacts of public information on individual drivers' decisions and on expectations formation, let us carry out the simulation experiments. Given a description of the route choice context, of which are specified in such a way as to bear simultaneously to actual conditions, the drivers independently supply decisions of route to destination. These decisions form the time-varying input function to a traffic simulator that yields the corresponding travel cost. Information on these consequences is subsequently provided to each driver. By controlling the type and amount of information supplied to the drivers, we can study the impacts of alternative information strategies on drivers' behavior as well as their expectations.

In what follows, we address to the question of whether route guidance information can convey substantially additional information to drivers even if drivers behave with rational expectations of their environment. Assume that at the beginning of period t, the public agent observes traffic volume at certain monitoring points on a network, and tries to forecast travel cost to be realized in period t. The major question for the public agent is how to forecast travel cost every point in time. The rational expectations are the stationary points to which all drivers' subjective expectations will converge in the long run. In the short run, the rational expectations are unknown to drivers. The same story goes for the public agent. Although the public agent is generally blessed with more informative environment than drivers, they are not in the position to know the exact states of rational expectations equilibria in the short run. Thus, the public agent should also learn how drivers subjective expectations will converge to the rational expectations. In what follows, the neural network modelling techniques are applied to design the mechanism by which the public agent can learn the final states of drivers' rational expectations.

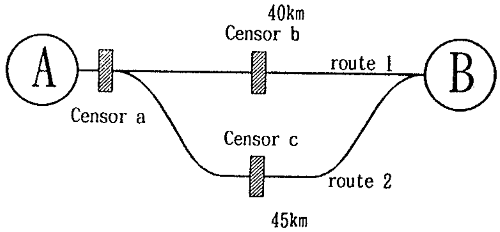

FIGURE 1. A Hypothetical Network for Numerical Examples

Let us first characterize the learning environment of the public agent. Denote the set of historical data observed at monitoring points up to this period by $\chi^t = \{x_i^k(i = 1, \cdots, n; k = t, t-1, \cdots)\}$. Given the set of historical data, the public agent forecasts the travel time $\tau(\chi^t) = \{\tau_a(\chi^t) : a \in \delta_s)\}$ based on χ^t. The forecasting mechanism is generally described by $\Gamma(\chi^t; \Theta)$, where Θ represent a forecasting model describing the relationship between the historical data and the travel time to be realized. Thus, the public agent can forecast the travel time by use of monitoring information χ^t and a forecasting model Θ. $\tau(\chi^t)$ need not coincide with the drivers' subjective or rational expectations, or both. Generally speaking, the public agent can have richer information than drivers, since drivers have no access to monitoring information. Thus, there exists information asymmetricity between the public agent and the drivers. This asymmetricity endows the public agent with the major informational advantage by which it can manipulate indirectly, in some ways, the drivers' route choices through public information.

The information providing rule Λ is the one which selects the message $\hat{e}$ to be informed to drivers based on the forecasting results $\{\Psi(\chi^t)\}$. The information providing rules can be described by a system $\hat{e} = \Lambda(\Psi(\chi^t, e); e \in \eta)$, where η is the set of messages and Λ is the selection rule. Let us consider, among others, the following rule: that to recommend the route to be chosen.

5.2 Description of Network Models

To simplify the experiments, we consider only one O-D pair connected by two routes with different characteristics as shown in Figure 1. Drivers are informed public messages at bifurcation point A. The drivers' initial expectations for travel cost of routes 1 and 2 are assumed to be homogeneous. They are described by normal distributions $N(50, 15)$. The private information ω_{a_s} is subject to a Weibull distribution $W(0, 10)$. The overall turbulence of travel cost by local traffic is subject to a certain type of stochastic process described later in the next chapter. It is assumed that 100 risk-neutral drivers are motivated to make simultaneous decisions in each iteration. The travel cost of both routes is varying over periods due to the fluctuation of local traffic and of all drivers' route choices. At the beginning of each period, the public agent can observe the local traffic volume of the period, but drivers cannot know it.

5.3 Neural network models

Artificial neural networks have been widely studied for information processing. Recently there has been also an increasing interest in application of neural network techniques to transportation engineering. Transportation application of neural networks modeling includes, namely, travel demand estimation, image processing, classification and pattern recognition and driver route choice analysis(Hai Yang. et al. 1993)[20]. It is generally recognized

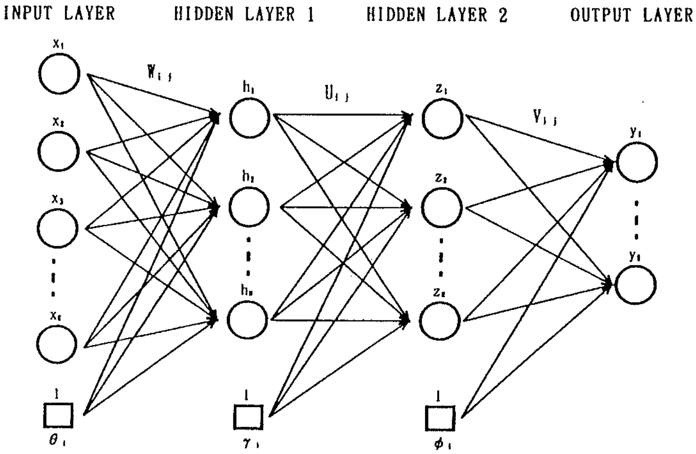

FIGURE 2. A Typical Multi-layer Feed-forward Neural network

that the neural network techniques have the capability to accommodate complicated problems without requiring explicit equations and/or correlating input/output data, generating reasonable results efficiently.

Figure 2. shows the connection scheme of a typical multi-layer feed-forward network. This network consists of processing elements arranged in four layers: an input layer, two hidden layers, and an output layer. The output of our neural model is the estimated travel time of each run. Given the estimated travel time, the public agent recommends drivers the route to be chosen. The input to the neural network is the travel time observed on both routes. The realized travel time which can be observed at the end of each period in time can be utilized as a teaching information in the learning procedure of the neural network model itself.

Let us summarize the basic scheme of our neural network modeling. Processing elements in adjacent layer are connected through connections W_{ij}, U_{ij}, and V_{ij}. The output emitted from each processing element is a function of the weighted outputs from the processing elements in the proceeding layer. Mathematically,

$$Y_j = F(X) = f[V_3 f\{U_2 f(W_1 + \theta) + \gamma\} + \phi] \tag{17}$$

where f is a nonlinear operator, W_1, U_2, and V_3 are weight matrices, and θ, γ, and ϕ are threshold vectors. Thus, it is possible to realize the X and y_j relationship by adjusting the weight matrices and threshold vectors.

The above equation can be written as three sub-equation in a neural network:

$$y_j = f(\sum_{i=1}^{M} V_{ij} z_i + \phi_j), \tag{18}$$

$$z_j = f(\sum_{i=1}^{N} U_{ij} h_i + \gamma_j), \tag{19}$$

$$h_j = f(\sum_{i=1}^{K} W_{ij} x_i + \theta_j) \tag{20}$$

where K, N and M are the number of neurons for the input, first hidden, and second hidden layers of the neural network, respectively: x_i is the ith element of input vector: W_{ij} is the weight of the interconnection between the ith element of input vector and the neuron j of the first hidden layer: U_{ij} is the weight of the interconnection between the neuron i of the first hidden layer and the neuron j of the second hidden layer, and V_{ij} is the weight of the interconnection between the neuron i of the second hidden layer and the neuron j of the output layer. Moreover, θ_j, γ_j and ϕ_j are the thresholds for the jth neuron of hidden layer one, hidden layer, and the output layer, respectively. $f(\cdot)$ is called an activation function which scales and smooths the output. Usually, a logistic function is used for this function. At the output layer, the error associated with the neuron j is

$$e_j = d_j - y_j \tag{21}$$

where d_j is the desired output(or "teacher signal") of the neuron j of output layer. Minimizing the sum of the squared errors:

$$E = \frac{1}{2}\sum_j (d_j - y_j)^2 \tag{22}$$

with respect to $W_{ij}, U_{ij}, V_{ij}, \phi_j, \gamma_j$ and and θ_j requires that the variables are moved in the descent direction of the objects function. According to the gradient decent method in optimization theory, these parameters should be modified as follows:

$$V_{ij}(n+1) = V_{ij}(n) + \Delta V_{ij}(n), \tag{23}$$

where:

$$\Delta V_{ij}(n) = \eta z_i \delta y_j + \alpha \Delta V_{ij}(n-1), \tag{24}$$

where:

$$\delta y_j = e_j y_j(1-y_j) = (d_j - y_j)y_j(1-y_j). \tag{25}$$

where η is a parameter of the learning rate, and $0 < \eta < 1$. α is the momentum gain which is used to speed up convergence and to restrain overshoot in the learning process, also $0 < \alpha < 1$. δ is the error, ΔV is change in weight and n is the cycle number. The threshold of the output layer is adjusted according to:

$$\phi_j(n+1) = \phi_j(n) + \Delta\phi_j(n), \tag{26}$$

where:

$$\Delta\phi_j(n) = \eta\delta y_j \cdot 1 + \alpha\delta\phi_j(n-1) \tag{27}$$

Adaption of the weight for hidden layer two neurons is then given by:

$$U_{ij}(n+1) = U_{ij}(n) + \Delta U_{ij}(n), \tag{28}$$

where:

$$\Delta U_{ij}(n) = \eta h_i \delta z_j + \alpha \Delta U_{ij}(n-1),$$
$$\delta z_j = z_k(1-z_j)\sum_{j=1}^{o} V_{ij}\delta y_j$$

where o is the number of neurons for output layer. The threshold of this hidden layer is adjusted according to:

$$\gamma_j(n+1) = \gamma_j(n) + \Delta\gamma_j(n), \tag{29}$$

where

$$\Delta\gamma_j(n) = \eta\delta z_j \cdot 1 + \alpha\delta\gamma_j(n+1) \tag{30}$$

Adaption of the weight for hidden layer one neurons is then given by:

$$W_{ij}(n+1) = W_{ij}(n) + \Delta W_{ij}(n), \tag{31}$$

where:

$$\begin{aligned} \Delta W_{ij}(n) &= \eta x_i \delta h_j + \alpha\Delta_{ij}(n-1) \\ \delta h_j &= h_j(1-h_j)\sum_{j=1}^{M} U_{ij}\delta z_j \end{aligned} \tag{32}$$

The threshold of this hidden layer is adjusted according to:

$$\theta_j(n+1) = \theta_j(n) + \Delta\theta_j(n), \tag{33}$$

where:

$$\Delta\theta_j(n) = \eta\delta h_j \cdot 1 + \alpha\delta\theta_j(n-1) \tag{34}$$

Training of the network starts with small random numbers assigned to all the weights and the thresholds. The training is terminated when either the maximum number of iterations is reached or the sum of squared output errors is reduced to an acceptable value. Figure 3. shows the training process with a back-propagation algorithm.

6. simulation experiments

Simulation is operated according to the following steps: (1) to construct the hypothetical neural network architecture; (2) to assume each driver's initial subjective expectation; (3) to generate local traffic volume by ARMA(2,2) model and Weibul random numbers associated with private information; (4) to calculate each driver's conditional expected utility for every message; (5) to determine the respective route that each driver may choose given a message; (6) to calculate the travel time on the routes by aggregating the results of individual route choices and using the three-layer, feed forward neural network with a back-propagation learning algorithm; (7) to calculate the conditional ex post utility given private information; (8) to aggregate for each message the conditional ex post conditional utility over all drivers; (9) to select the message which maximizes the aggregated ex post utilities; (10) to determine the driver's choices in this iteration; (11) to up-date each driver's subjective expectation;

As mentioned earlier, one objective of this paper is to construct a neural network model for predicting traveling time on a network where route navigation information is provided. Using the neural network, the public agent tries to forecast the travel time to be realized. The neural network used in this paper consists of an input layer, a hidden layer and a output layer as shown in Figure 4. There are 4 processing elements in the input layer,

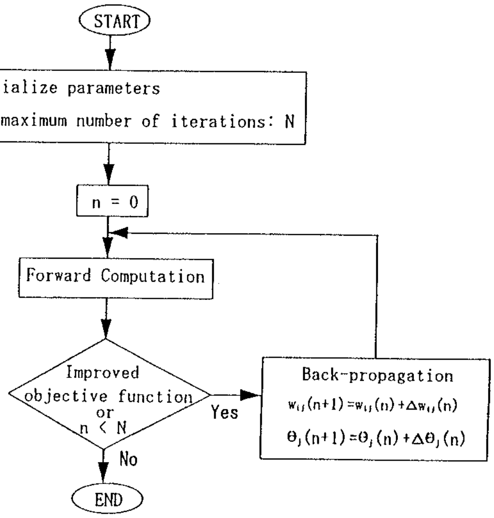

FIGURE 3. Flow Chart of a Back-propagation Training Algorithm.

these are the local traffic volume of route 1 and 2 and through traffic volume of route 1 and 2, respectively. All the values taken in input processing elements will be normalized into values to be between 0 and 1 using a logistic function and then transmitted to the hidden layer in the neural network. Two processing elements in the output layer are used to indicate a predicted travel time of route 1 and 2.

Before examining route navigation in detail, we report some results of the validation experiments on the performance of the neural network model. The number of neurons used in the experiments ranged from 1 to 4 for the first layer and from 1 to 4 for second hidden layer.

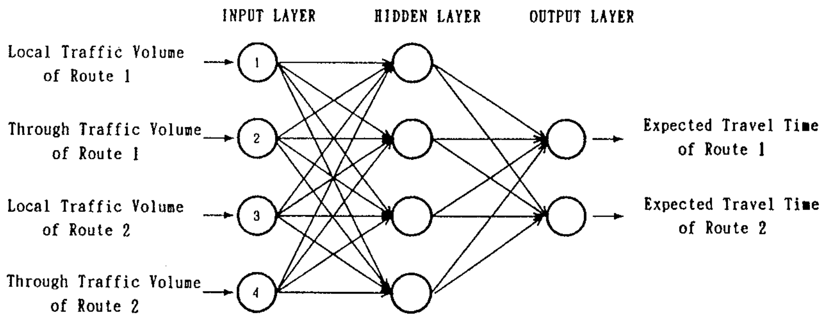

FIGURE 4. Neural Network Model for Route Navigation

Table 1a presents the results of the sum of squared learning errors with the different numbers of the hidden element(s) from 1 to 4 and where 3*3 hidden elements stands for 3 elements in the first hidden layer and 3 elements in the second hidden layer. Table 1b shows the difference of the sum of squared errors with the different values of learning rate for momentum gain $\alpha = 0.8$. In this test, the difference of the hidden layer had little impact but the networks with 1 and $3*3$ hidden elements indicated a limitation. In contrast, we observed that the value of learning rate had large impact on the performance of the network as shown in Table 1b, but the initial value of the sum of squared of errors are very similar.

TABLE 1a. Changes of Sum of Squared Errors across the Number of Hidden elements for α 0.8

Learning rate	Hidden elements	S.S.E.* (5000 iteration)	Iteration (S.S.E.=0.009)
0.9	1	0.00956	I.P.**
	2	0.00553	1083
	3	0.00550	450
	4	0.00548	310
	3×3	0.00956	I.P.

*S.S.E.: Sum of Squared Errors **I.P.: Impossible

TABLE 1b. Changes of Sum of Squared Errors across the Value of Learning rate for α 0.8

Learning rate	S.S.E.		Iteration
	Initial value	5000 iteration	(S.S.E.=0.009)
0.01	0.287	0.01500	13740
0.1	0.265	0.00571	2480
0.9	0.277	0.00548	310

Through this test, the optimum configuration of the neural network architecture was determined. Learning rate and momentum gain were set to be 0.9 and 0.8 respectively for all simulation experiments. Moreover, the number of processing elements in the hidden layer was fixed as 4 as shown in Figure 4. Also, the overall turbulence of travel time by local traffic is given by a time series model. A time series is a set of observations that are arranged chronologically. If a process consists of both AR(Autoregressive) and MA(Moving Average) parameters, it is called an ARMA process. When there is one AR and one MA parameter the ARMA process is denoted as ARMA(1,1) and the equation for this process is

$$(Z_t - \mu) - \phi_1(Z_{t-1} - \mu) = \alpha_t - \theta_1\alpha_{t-1} \tag{35}$$

where μ is the mean level of the process, ϕ_1 is the nonseasonal AR parameter, α_t is the white noise term at time t that is identically independently distributed(IID) with a mean of 0 and variance of σ_α^2 [i.e. IID($0,\sigma_\alpha^2$)]. By utilizing the B operator (backward shift operator B), the ARMA(1,1) processor can be equivalently written as

$$(1 - \phi_1 B)(Z_t - \mu) = (1 - \theta_1 B)\alpha_t$$

or

$$\phi(B)(Z_t - \mu) = \theta(B)\alpha_t \tag{36}$$

where $\phi(B) = (1 - \phi_1 B)$ and $\theta(B) = 1 - \theta_1(B)$ are, respectively, the AR and MA operators of order one. For example, with p AR parameters and q MA parameters, the ARMA processor is denoted by ARMA(p,q) and is written as

$$\begin{aligned}(Z_t - \mu) - \phi_1(Z_{t-1} - \mu) - \phi_2(Z_{t-2} - \mu) - \cdots - \phi_p(Z_{t-p} - \mu)\phi_1 \\ = \alpha_t - \theta_1\alpha_{t-1} - \theta_2\alpha_{t-2} - \cdots - \theta_q\alpha_{t-q}\end{aligned} \tag{37}$$

By implementing the B operator, equation(37) can be presented more conveniently as

$$(1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p)(Z_t-\mu)=(1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q)\alpha_t$$

or

$$\phi(B)(Z_t-\mu)=\theta(B)\alpha_t \tag{38}$$

where $\theta(B)=1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p$ is the AR operator p and $\theta(B)=1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q$ is the MA operator of order q. In this paper, the white noise term α_t was set at a mean of 0 with a variance of 2. In equation(37), the mean level of the process μ was determined to be 15. The nonseasonal parameters π_1, π_2 set to be 0.7, 0.2 respectively. Moreover, θ_1,θ_2 was set to be 0.7, 0.2 respectively.

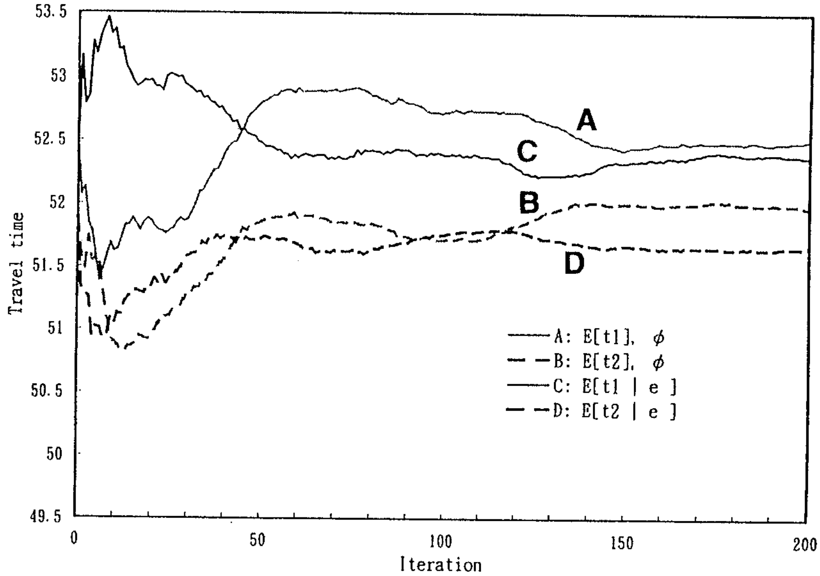

FIGURE 5. Public Information and Learning Procedures

In this paper, two experimental cases are conducted. In case 1, the predicted travel time are provided at point A. In contrast, no public information is provided in case 2. Figure 5 illustrates the impacts of information on the objective distributions of travel time. The public agent informed the drivers with the message 'Choose route 1' or 'Choose route 2'. For each sample period, we calculated the respective average of travel time which had been observed up to the concerned period. Figure 5 also shows the changing patterns of the calculated means of travel time. In this figure, $E[t_1],\phi$ and $E[t_2],\phi$ represents the objective distribution of travel time of route 1 and 2 when no public information is provided. In contrast, $E[t_1 \mid e]$ and $E[t_2 \mid e]$ are the objective distribution of travel time when public information is provided. Also, we know that the travel time of both routes for $E[t_1 \mid e]$ and $E[t_2 \mid e]$ decrease compared to the cases when no public information is provided.

Thus, as far as our simulation is concerned, public information is not neutral and conveys substantial additional information to the drivers.

Figure 6 shows the changing patterns of expectations for objective distributions of travel time of routes 1 and 2. In this figure, $E[t_1]$ and $E[t_2]$ stands for the expectations of objective distribution of travel time of route 1 and

2. $E[t1 \mid e=1]$ and $E[t_2 \mid e=1]$ shows conditional expectations on message ($e=1$) for objective distribution of travel time of route 1 and 2, respectively. $E[t_1 \mid e=2]$ and $E[t_2 \mid e=2]$ shows conditional expectations on message ($e=2$) for objective distribution of travel time of route 1 and 2, respectively.

From this figure, when the message indicates $e=1$ or 2, the conditional expectations of the objective distribution of route 1 converge upon different values.

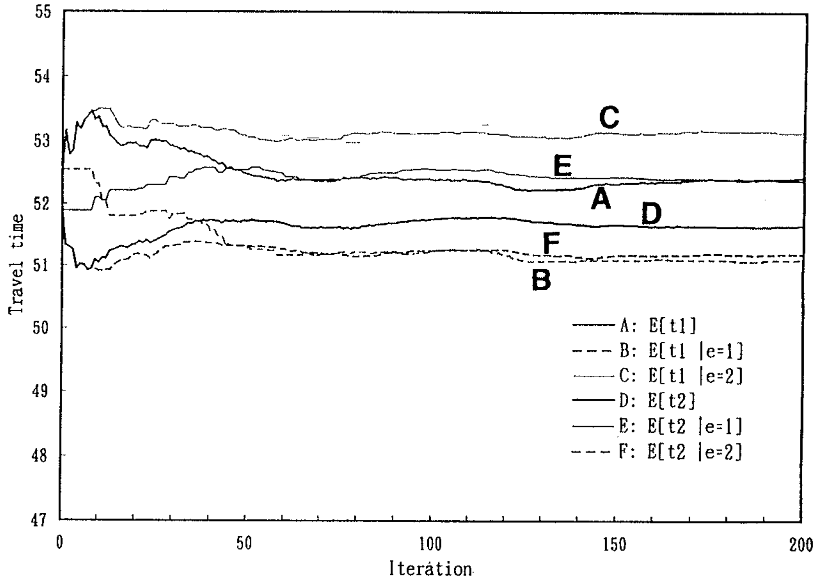

FIGURE 6. Changing of Objective Expectations of Travel time

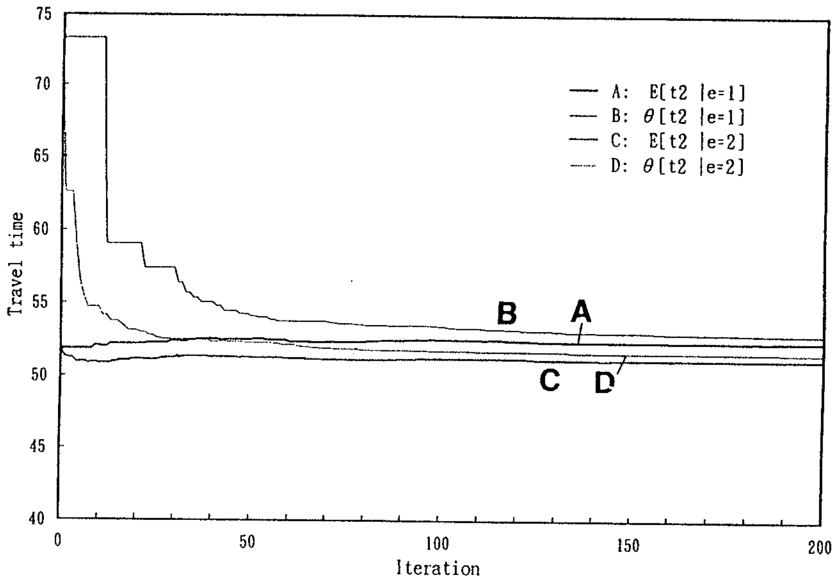

FIGURE 7. Formation of the Process of Rational Expectations by the Learning Procedures

Simulation was continued over 200 periods. Figure 7 shows that each drivers' process of rational expectations forms objective distributions and subjective distributions of travel time of route 2 by the learning procedures. The drivers' subjective beliefs on travel time converge upon the rational expectations by around the 200-th period as shown in this figure.

7. Conclusion

This paper presented a neural network model to predict traveling time on networks and to investigate the impacts of route guidance systems upon driver's long-term behavior. Through experiments, the optimal configuration of the neural network architecture is determined., There are 4 processing elements in the input layer, 4 processing elements in the hidden layer and 2 processing elements in the output layer to indicate a predicted travel time of route 1 and 2. Also, the objective distributions and subjective distributions of travel time of routes forms rational expectations by the repeated learning. More work is needed to enlarge the scope of the study and to explore more deeply the drivers' behavior with rational expectations under incomplete and decentralized information.

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