Effects of Local and Global Degrees of Freedom on Load-Displacement Behavior of Soil Ground in FE Analysis

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The problem of loading through a rigid strip footing on a elastic soil ground are analyzed with the finite element method. Effects of the number of DOF on the load-displacement are investigated. Two ways for increasing the number of DOF in the system were examined: one is to increase the number of elements and another is to increase the number of nodes per element.

It is shown that, more the number of degrees of freedom is, lower the load at a particular displacement is. The load reduction effect associated with the increase in local degrees of freedom is slight when a fine mesh is used. For the problem treated here the use of small elements of low order, for instance linear, is recommended accordingly.

Key words : Finite Element Method, Degrees of Freedom, Elastic Soil Ground, Load-Displacement Relation

1. Introduction

The solution for an unknown in finite element analysis is expected to be improved with the increase in the number of degrees of freedom (DOF) with respect to the unknown. The increase in the number of DOF can be achieved in two ways: one is to increase the number of elements and the other is to increase the number of nodes per element. In the former way the number of DOF is increased globally but not locally. In the latter way it is increased locally and therefore globally.

It will be discussed which is better to use fine meshes of low order elements or to use relatively coarse meshes of high order elements in the examined problem.

In this study, the problem that the load is applied through a rigid strip footing to an elastic soil ground is treated. On applying two ways mentioned above, the results will be discussed through load-displacement relations obtained from finite elements analyses.

Elements of high orders up to 4 are used in this study. The difficulty in the formulation of interpolation functions and the numerical integration increases as the order of interpolation functions increases. It will be shown that the difficulty can be overcome by applying approved techniques in literatures.[1],[2],[3]

2. Method for analysis

2.1 Elements

Four types of elements were used to investigate the effects of local degrees of freedom on the results (see Table 1 and Fig.1). All the types of elements are triangular and geometrically linear; interpolation functions are consisted of complete polynomials.

2.2 Interpolation functions

Area-coordinates (L_1 L_2 L_3) were adopted for local coordinates for the convenience of the numerical integration. The *ad hoc* formulation of interpolation functions for each type of element is possible for lower order elements. In facts for T3 elements we have the following nodal approximation:

$$u = \langle N \rangle \{\bar{u}\} \qquad \text{--------------------(1)}$$

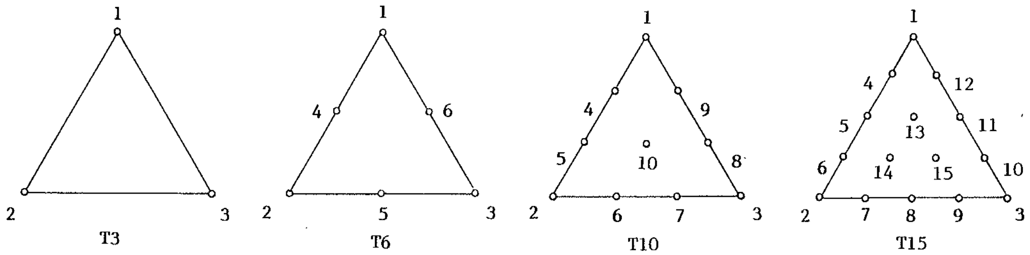

Fig.1: Element types used in this study.

Table 1: Element types and characteristics for interpolation and numerical integration.

Element type	T3	T6	T10	T15
Number of geometrical nodes	3	3	3	3
Order of interpolation functions	1	2	3	4
Number of interpolation nodes	3	6	10	15
Order of integrands in the stiffness matrix	0	2	4	6
Number of sampling points for numerical integration	1	3	7	12

Note: Only T3 elements are isoparametric; others are subparametric.

where

$\langle N\rangle = \langle N_1 \; N_2 \; N_3\rangle$ with $N_1=L_1$, $N_2=L_2$, and $N_3=L_3$

and

$\{\bar{u}\} = \langle \bar{u}1 \; \bar{u}2 \; \bar{u}3\rangle^T$

N_is (i=1,2,3) are interpolation functions for linear nodal approximation. $\{\bar{u}\}$ is the vector of nodal values of the unknown u.

For higher order elements we can adopt the method, shown by Zienkiewicz[1)], for the formulation of interpolation functions. Referring to Fig.2, the problem is to find $N_i^{(n+1)}$ when $N_i^{(n)}$ is given, where $N_i^{(n)}$ is the interpolation function corresponding to the i-th node in n-th order elements($n>=1$). To solve the problem we use the following properties: for an i-th node, being on the side, say, 12 of the element of (n+1) order,

i) $N_i^{(n)} = 1$; and $N_j^{(n)} = 0$ for $j \neq i$

and

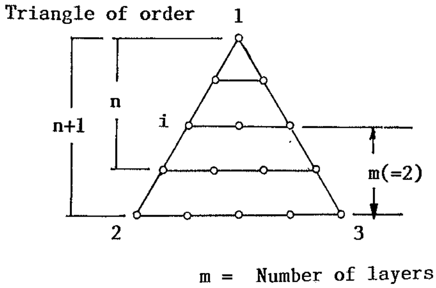

Fig.2: Explanation for the formulation of interpolation functions for high order triangular elements.

ii) $L_1(n+1) = m/(n+1)$

where m is the number of layers lying under the i-th node.

We can obtain the interpolation functions for i-th node such as:

$$N_i^{(n+1)} = c.L_1^{(n+1)}.N_i^{(n)};\ c = (n+1)/m \qquad \text{-----(2)}$$

If the node i is on the side 13, L_1 should be read as L_2, and if on the side 23, L_1 should be changed to L_3, respectively in the Eq.(3). By this way we obtain interpolation functions for T6, T10 and T15 elements. They are listed in the appendix.

2.3 Numerical integration

In calculating a stiffness matrix, the integration over an element must be performed. The integration is simply done in the analytical way for linear and quadratic elements; however for higher order elements it is rather complicated. For such elements the element of reference should be used for the simplicity of the expressions and numerical integration techniques should be used instead of the analytical way.

A triangular reference element can be made in the space composed of two independent variables, say L_1 and L_2, of three area-coordinates (L_1, L_2, L_3). A transformation from (L_1, L_2, L_3) to global coordinates (x,y) will realize the transformation of integration in real space to that in the reference element. When the transformation is linear, we have

$$x = \langle L_1\ L_2\ L_3\rangle\{\bar{x}\};\ y = \langle L_1\ L_2\ L_3\rangle\{\bar{y}\} \qquad \text{-------------------- (3)}$$

with

$$\{\bar{x}\}=\langle\bar{x}_1\ \bar{x}_2\ \bar{x}_3\rangle^T;\ \{\bar{y}\}=\langle\bar{y}_1\ \bar{y}_2\ \bar{y}_3\rangle^T$$

where $\{\bar{x}\}$ and $\{\bar{y}\}$ are values of x and y coordinates of geometrical nodes.

For the numerical integration we can use some quadrature formulas. The formulas in which the symmetry of the configuration of sampling points as well as the prescribed accuracy are assured must be used. In this study the formulas presented by Cowper[2] and Laursen and Gellert[3] were used.

The least number of integration or sampling points can be determined by accounting for the highest order of polynomials appearing in integrands in elements of the stiffness matrix. The order can be simply determined when the geometrical transformation is linear as Eq.(3) because the determinant of the Jacobian matrix becomes constant in this case. The least number of sampling points were determined according to Laursen and Gellert[3]. The correspondence between the types of element and the number of sampling points is given in Table 1. Values of area coordinates of sampling points and corresponding weights are listed in literature[3].

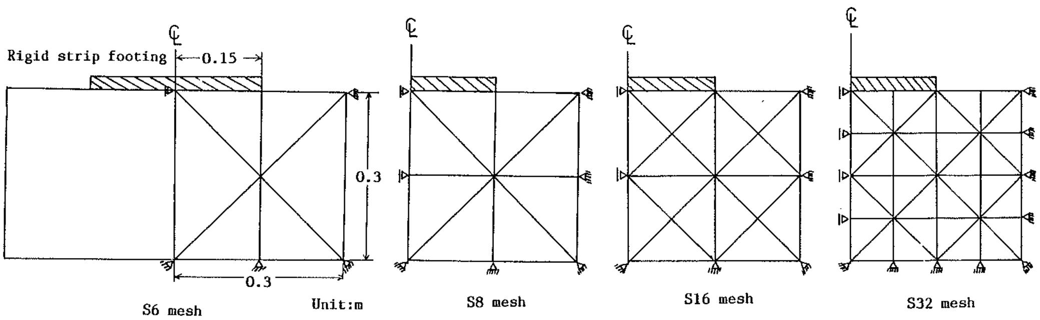

Fig.3: Small model and elements configurations used.

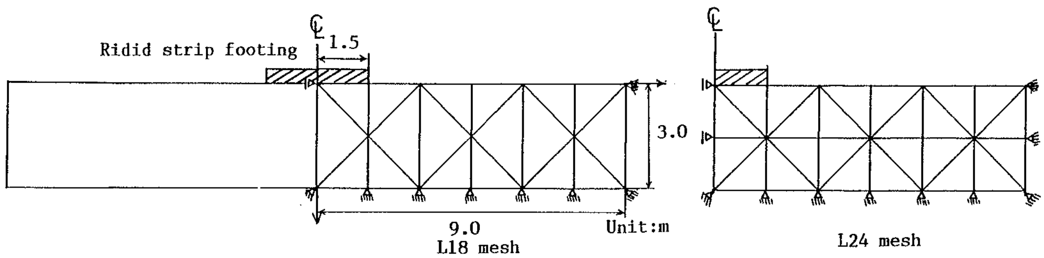

Fig.4: Large model and elements configurations used.

2.4 Model ground for analyses

The plane strain condition was assumed. Two types of model grounds shown in Figs.3 and 4 were analyzed. The domain of the first type of ground is small because those model tests were supposed in which load would be applied through a rigid footing. This type of model will be called the S model. The second is for realizing a more realistic condition in which a sand layer is spread infinitely and the depth of the layer is finite. The model will be called the L model.

For each type of model ground, different mesh configurations were used in which the total number of elements is different. The numbers of elements are 6,8, 16 and 32 for the S model ground. The mesh configurations are distinguished by the notation S6, S8, S16 and S32 for 6, 8, 16 and 32 elements. Similarly, for the L model, two types of mesh were analyzed: L18 and L24 meshes.

In an analysis on each type of mesh configurations, 4 types of elements described above were used. The number of analysis runs becomes 16 for the S model and 8 for the L model.

2.5 Constitutive model of the soil

The soil of the model ground was assumed to be linearly elastic. Elastic constants were not varied. The values for elastic constants and some parameters used in analyses are listed in Table 2.

Table 2: Values of parameters used in analyses

Young's modulus E (tf/m^2)	$3x10^3$
Poisson's ratio ν	0.47
Unit weight of soil γ (tf/m^3)	1.6
Pressure coefficient at rest K_0	0.55

3. Results

3.1 Load-displacement relations

Load-displacement relations resulted from finite element analyses will be shown and discussed. Figs.5 and 6 show examples of the results for the S model and for the L model, respectively. In each figure, the effects of the number of nodes in an element are investigated. We see that load-displacement curves are steeper and accordingly the load at a particular displacement is larger as the number of nodes per element is less.

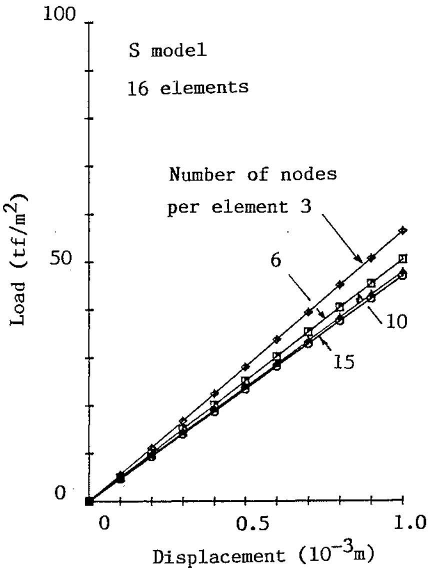

Fig.5: Load-displacement relations for S model of 16 elements.

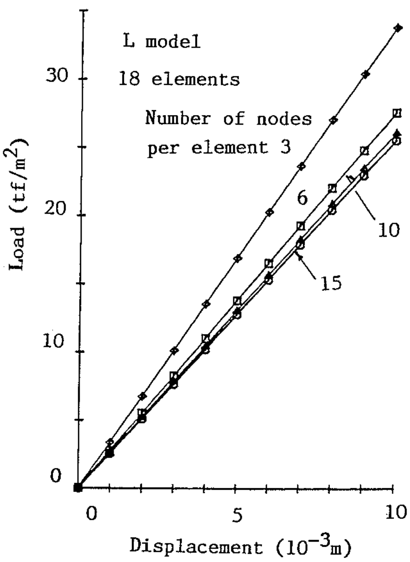

Fig.6: Load-displacement relations for L model of 18 elements.

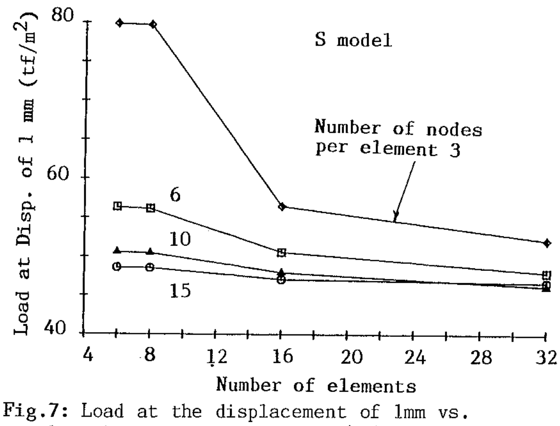

Fig.7: Load at the displacement of 1mm vs. total number of elements for the S model.

3.2 Effects of local DOF

In Fig.7, the load at the displacement of 1mm is plotted against the number of elements for the S model. We can see in this figure that, for any type of mesh, i.e. for any number of elements, more the number of nodes per element is, smaller the load is. Such an effect of the local number of degrees of freedom tends to be less with the increase in the number of

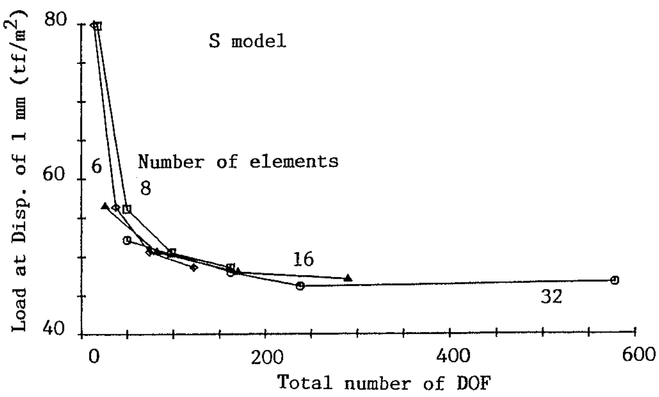

Fig.8: Load at the displacement of 1mm vs. total number of degrees of freedom for the S model.

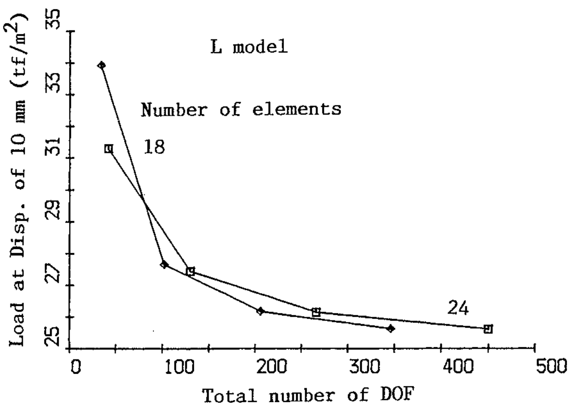

Fig.9: Load at the displacement of 10mm vs. total number of degrees of freedom for the L model.

elements; for instance, when the number of elements is 32, the effect is slight and the load for T10 elements can be even less than that for T15 elements.

3.3 Effects of total number of DOF

In Figs.8 and 9, the load at a particular value of the displaceement is plotted against the total number of degrees of freedom in the system for the S model and for the L model, respectively. The particular values of displacement are 1mm for the S model and 10mm for the L model.

We can see in these figures the effects of global degrees of freedom.

The load is reduced by approximately 50% for the S model and 30% for the L model as the number of degrees of freedom increases up to the examined maximum value.

Figs.8 and 9 also show that the effect of the load reduction can be made to the almost same extent by using higher order elements and by increasing the number of elements.

4. Discussion

In the design of bearing capacity of a strip footing on a sand layer, a safe or conservative design is such that a smaller bearing capacity is estimated. If the bearing capacity is overestimated, the design will be unsafe. From this point of view, we can say with the consideration of results shown in Figs.8 and 9 that the solution can be improved by increasing the total number of degrees of freedom in finite element analyses.

In two ways for increasing the total number of degrees of freedom in the system, the way in which the number of elements is increased is effective because the load reduction effect associated with the higher order elements is less for finer meshes.

5. Conclusions

The problem of loading through a rigid strip footing on a elastic soil ground was analyzed with the finite element method. Effects of the number of DOF on the load-displacement were investigated. Two ways for increasing the number of DOF in the system were examined: one is to increase the number of elements and another is to increase the number of nodes per element.

For a particular number of elements, the increase in the number of nodes per element resulted in lower values of load at a certain displacement. Similarly, for a particular type of element, the load became lower when the number of elements was increased. The load reduction effect associated with the increase in the number of nodes per element was less than that associated with the increase in the number of elements. In other words, the load reduction effect when the order of an element is made higher is slight for fine meshes.

It is accordingly concluded that, for the problem treated in this study, the use of small elements of low order is recommended. It is also supported because of the simplicity of programing and the accuracy in integration.

The errors essentially contained in finite element solutions was not treated in this paper. The estimation and evaluation of the errors and improvement of the solutions have been an important subject in the field of FE analyses. An approach is developed by the first author elsewhere[4].

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Appendix: Interpolation functions for higher order elements

(1)T6 element

$N_1=L_1(2L_1-1)$ $N_2=L_2(2L_2-1)$ $N_3=L_3(2L_3-1)$ $N_4=4L_1L_2$ $N_5=4L_2L_3$

$N_6=4L_1L_3$

(2)T10 element

$N_1=(3L_1-1)(3L_1-2)L_1/2$ $N_2=(3L_2-1)(3L_2-2)L_2/2$ $N_3=(3L_3-1)(3L_3-2)L_3/2$

$N_4=9L_1L_2(3L_1-1)/2$ $N_5=9L_1L_2(3L_2-1)/2$ $N_6=9L_2L_3(3L_2-1)/2$

$N_7=9L_2L_3(3L_3-1)/2$ $N_8=9L_3L_1(3L_3-1)/2$ $N_9=9L_1L_3(3L_1-1)/2$

$N_{10}=27L_1L_2L_3$

(3)T15 element

$N_1=L_1(4L_1-1)(4L_1-2)(4L_1-3)$ $N_2=L_2(4L_2-1)(4L_2-2)(4L_2-3)$

$N_3=L_3(4L_3-1)(4L_3-2)(4L_3-3)$ $N_4=8L_1(4L_1-1)(4L_1-2)L_2/3$

$N_5=4L_1L_2(4L_1-1)(4L_2-1)$ $N_6=8L_1(4L_2-1)(4L_2-2)L_2/3$

$N_7=8L_2(4L_2-1)(4L_2-2)L_3/3$ $N_8=4L_2L_3(4L_2-1)(4L_3-1)$

$N_9=8L_2(4L_3-1)(4L_3-2)L_3/3$ $N_{10}=8L_3(4L_3-1)(4L_3-2)L_1/3$

$N_{11}=4L_3L_1(4L_3-1)(4L_1-1)$ $N_{12}=8L_3(4L_1-1)(4L_1-2)L_1/3$

$N_{13}=32L_1L_2L_3(4L_1-1)$ $N_{14}=32L_1L_2L_3(4L_2-1)$

$N_{15}=32L_1L_2L_3(4L_3-1)$