Properties of Circulation in Lake Koyamaike

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The wind driven circulation in Lake koyamaike is simulated by means of the finite element method and its result is compared with the observed data. In the simulation, the distribution of the wind velocity at the water surface and the effect of the islands in the lake are considered, and the characteristics of the circulation in the lake are clarified to some extent.

1. Introduction

Wind driven circulation in Lake Koyamaike which has five islands is studied in a numerical investigation, using the finite element method. Koyamaike is so shallow that the Ekman-type model can be applied for the analysis of its circulation. Numerical studies about shallow lake circulation, using the finite element method, have been done by Liggett et al.[1),2),3),4)], Cheng[5)], Yokoshi & Tomidokoro[6)] and Muraoka & Fukushima[7)].

Liggett et al. modified the equation of water motion, using the Ekman-type model which can be applied for shallow water lake and solved the equation numerically by means of the finite element method under the following approximations and restrictions :

1. $D/L \ll 1$ and $D/d \ll 1$ in which D and L are typical vertical and horizontal dimensions and $d=\pi\sqrt{2\eta/f}$ (the Ekman depth of friction influence) in which η is the vertical eddy viscosity and f is the Coriolis parameter.
2. Nonlinear terms and horizontal diffusion of momentum are neglected.
3. The Coriolis parameter is constant and the eddy viscosity is constant from the water surface to the bottom.
4. Vertical velocities are small as compared to horizontal velocities (a consequence of shallow water).

In the calculation, the surface shear stress which is assumed to be induced by wind was given uniformly. But in the case of rather small lakes like Lake Koyamaike, if they are surrounded by mountains or have large islands, the characteristics of wind may be different at different places and the water surface shear stress can not be uniform.

In this paper, the numerical method presented by Liggett et al. is applied for the analysis

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of the circulation in Lake Koyamaike, modifying the distribution of the water surface shear stress. Also some observed data about the circulation are presented.

2. Summary of Lake Koyamaike

The topography of Lake Koyamaike, which is located in lat. 35°30′ North and in long. 134° 09′ West, is shown in **Fig. 1**. The surface area of the lake is 6.9km^2, the length of its shore line is 17.5km and the mean water depth is about 3.5m. It has five islands, the largest one is 0.2km^2 in area. Although the area north-east of the lake is flat and open, the south-west half of the lake is surrounded by mountains which weaken the wind from the west or south. A lot of small rivers flow into the lake but River Koyamagawa is the only river which starts from the Lake. The discharge of the river is very small except during flood which occurs usually in June, July, September and October. It is reported that sludge is settled at the bottom of the lake, forming a layer with about 40cm of maximum thickness. When the wind blows with a speed more than 3m/sec, the sludge layer is disturbed by the lake ciculation and the color of the water turns to yellow due to the suspension of very fine materials.

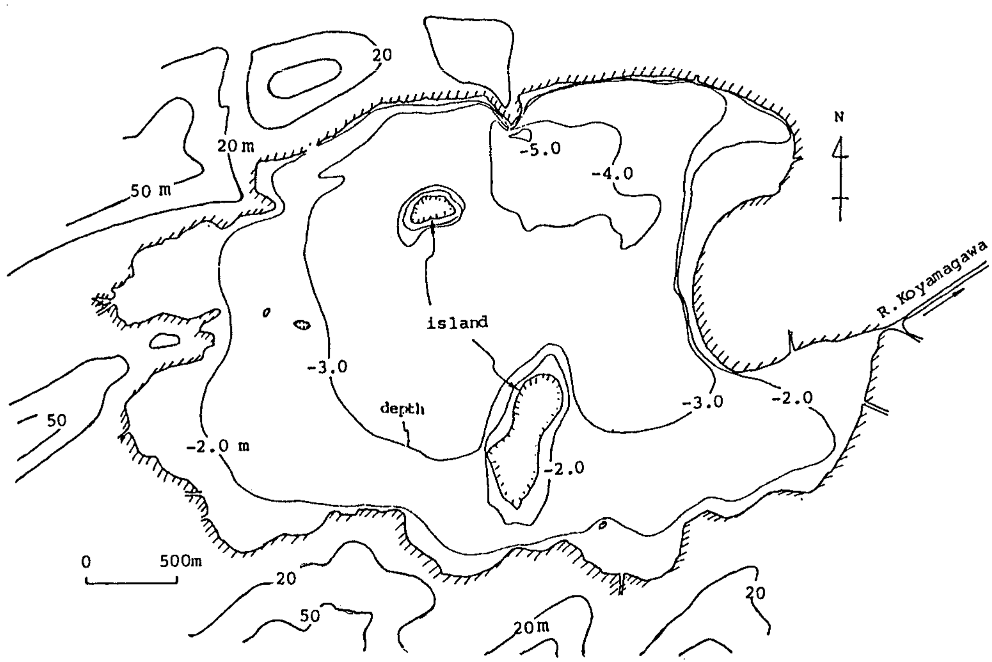

Fig. 1 Topography of Lake Koyamaike.

3. Numerical Analysis

3. 1 Basic Equation

With the approximations and restrictions mentioned in the introduction, the unsteady linearlized equations of motion are simplified as :

$$\frac{\partial u}{\partial t}-fv=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\eta\frac{\partial^2 u}{\partial z^2} \quad \cdots\cdots(1)$$

$$\frac{\partial v}{\partial t}+fu=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\eta\frac{\partial^2 v}{\partial z^2} \quad \cdots\cdots(2)$$

$$g=-\frac{1}{\rho}\frac{\partial p}{\partial z} \quad \cdots\cdots(3)$$

and the equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad \cdots\cdots(4)$$

in which u, v and w : the velocity components in x (positive eastward), y (positive northward) and z (positive upward and zero at the surface) directions, t : time, p : the local pressure, f : the Coriolis parameter, ρ : the fluid density, η : the eddy viscosity, g : the acceleration due to gravity.

These four equations should be solved with the following conditions,

on solid boundaries $(z=-h)$ $\quad u=v=w=0$ $\cdots\cdots(5)$

at the free surface $(z=0)$ $\quad \tau_x=\eta\frac{\partial u}{\partial z},\quad \tau_y=\eta\frac{\partial v}{\partial z}$ $\cdots\cdots(6)$

in which τ_x andτ_y are the wind stress functions in x and y directions respectively and h is the local flow depth.

The terms on time derivatives of the equations are removed by means of the Laplace transform such as

$$\tilde{u}=\int_0^\infty ue^{-st}dt \quad \cdots\cdots(7)$$

After all the equations multiplied by e^{-st} are integrated, transformed velocities u and v can be solved using transform eqs. (1) and (2). Then the vertical average of the transformed velocities

$$\bar{u}=-\frac{1}{h}\int_{-h}^{0}\tilde{u}\,dz,\qquad \bar{v}=\frac{1}{h}\int_{-h}^{0}\tilde{v}\,dz \quad \cdots\cdots(8)$$

are obtained, which include $\partial\tilde{P}/\partial x$ and $\partial\tilde{P}/\partial y$. When the stream function ψ defined as

$$\bar{u}=-\frac{1}{h}\frac{\partial\tilde{\psi}}{\partial y}, \qquad \bar{v}=\frac{1}{h}\frac{\partial\tilde{\psi}}{\partial x} \quad \cdots\cdots(9)$$

is introduced, the integrated continuity equation of eq. (4) is automatically satisfied. By substituting eq. (9) into eq. (8), $\partial\tilde{P}/\partial x$ and $\partial\tilde{P}/\partial y$ are solved as functions of ψ. And by cross differentiation of $\partial\tilde{P}/\partial x$ and $\partial\tilde{P}/\partial y$, the following equation can be obtained :

$$\frac{\partial^2\tilde{\psi}_*}{\partial x_*^2}+\frac{\partial^2\tilde{\psi}_*}{\partial y_*^2}+A(x_*, y_*;s_*)\frac{\partial\tilde{\psi}_*}{\partial x_*}+B(x_*, y_*;s_*)\frac{\partial\tilde{\psi}_*}{\partial y_*}+C(x_*, y_*;s_*)=0 \quad \cdots\cdots(10)$$

in which sufix $*$ denotes nondimensional variables and the nondimensional parameters are defined as

$$\left.\begin{array}{l} x_*=x/L,\ y_*=y/L,\ z_*=z/L,\ t_*=ft,\ u_*=(fL/gD)u, \\ v_*=(fL/gD)v,\ w_*=(fL^2/gD^2)w,\ h_*=h/D,\ p_*=(p/\rho gD)+(z/D), \\ \Delta_*=(fL\tau_*/\eta g)\ \text{and}\ \Gamma_*=(fL\tau_y/\eta g). \end{array}\right\} \quad \cdots\cdots(11)$$

And A, B and C are functions of x_*, y_* and s and their forms are given in appendix II.

$\psi(s)$ in eq. (10) can be solved numerically and transformed into $\psi(t)$ by means of the collocation method of Schapery[8] which is used for the numerical Laplce transform inversion.

3. 2 Circulation in Lake Koyamaike

In order to analyze the wind driven circulation in Lake Koyamaike which has five islands, eq. (10) is solved numerically by means of the finite element method. As the lateral boundary condition for eq. (10), slip and non-slip conditions can be considered. In the case of the slip condition, the tangential velocity u_s is allowed but the normal velocity u_n is zero, whereas in the case of the non-slip condition, both of u_s and u_n are zero. In this paper, we use the slip condition that the mean velocity at the lateral boundary must be tangential to the boundary. This condition is expressed by ψ as follows :

$$\left.\begin{array}{ll} \psi=0 & \text{on}\ \lambda_o \\ \psi=C_i & \text{on}\ \lambda_i,\quad i=1, 2, 3, 4, 5 \end{array}\right\} \quad \cdots\cdots(12)$$

where λ_o is the lateral boundary of the lake, and λ_i is the lateral boundary of the ith island. The constant values of C_i's are to be determined in the course of solving the boundary problem governed by eq. (10) and eq. (11). In this paper, the method of C_i determination proposed by Cheng[5] is used.

The finite element display of the lake is shown in **Fig. 2** in which the number of the finite elements is 554.

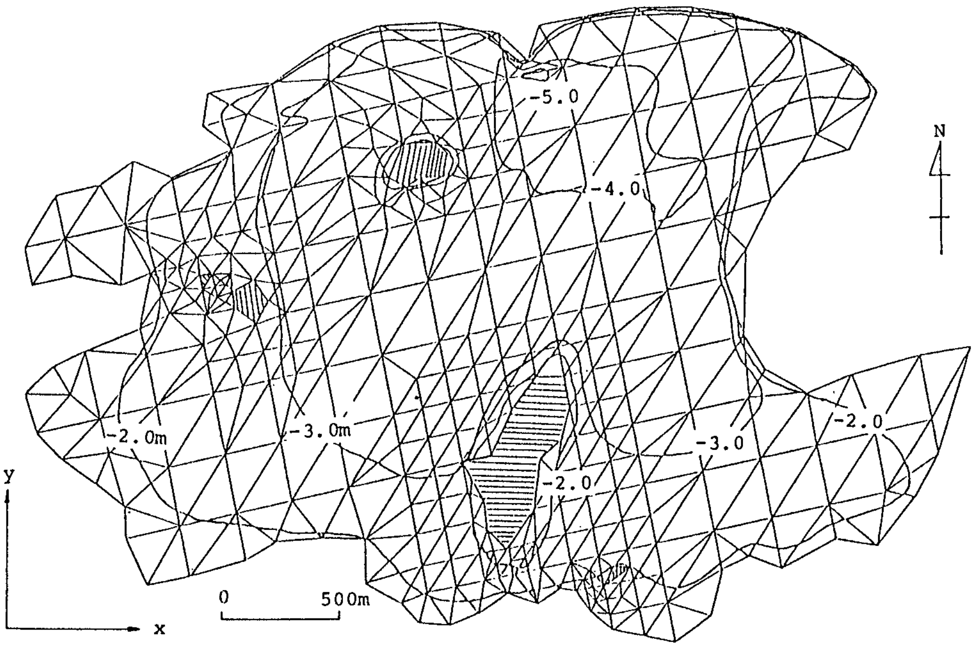

Fig. 2 Finite element display of Lake Koyamaike.

The surface shear stress τ_{wind}, which is divided into τ_x and τ_y, is given by

$$\tau_{wind}=\rho_a \cdot C_f \cdot U_z^2 \quad \cdots\cdots (13)$$

in which ρ_a is the air density and U_z is the wind velocity 10m above the water surface and C_f is the coefficient of wind friction. Deacon & Webb[9] estimated C_f by

$$C_f=(a_1+b_1U_z)\times 10^{-3} \quad \cdots\cdots (13)$$

where $a_1=1.00$, $b_1=0.07$ and the dimension of U_z is m/sec.

The eddy viscosity η is assumed to be constant from the water surface to the bottom and calculated by

$$\eta=0.00056\sqrt{C_f}\cdot D\cdot U_z \quad \cdots\cdots (15)$$

which was proposed by Yokoshi & Tomidokoro[6], in which D is the mean depth. The Coriolis parameter f at the lake is 0.0000845 (1/sec).

The circulation of the steady state is calculated for the north-west wind which occurs mainly in winter. The distribution of the wind velocity at the water surface is given as in **Fig. 3**, considering the height and scale of the

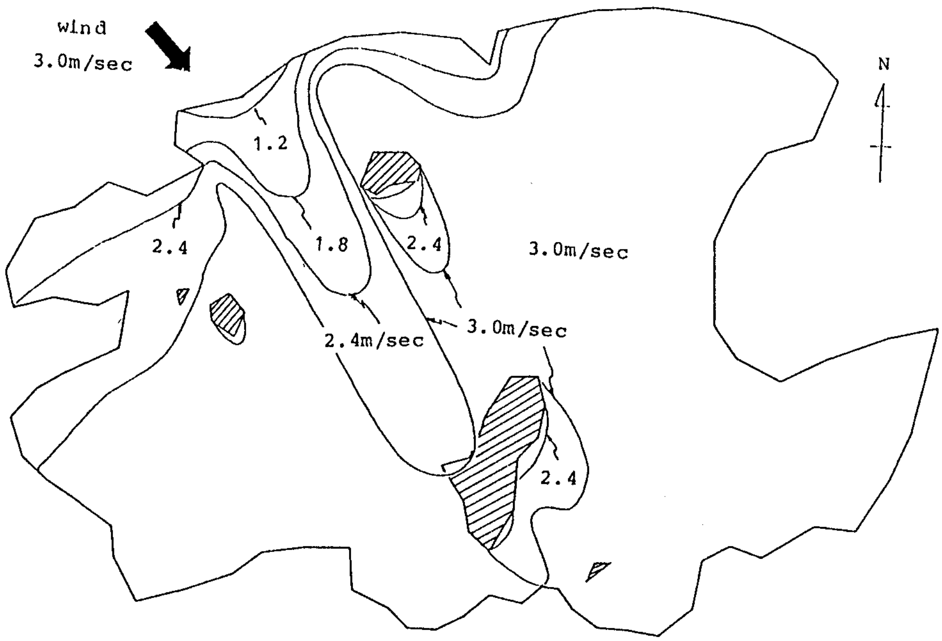

Fig. 3 Distribution of the wind velocity at the water surface.

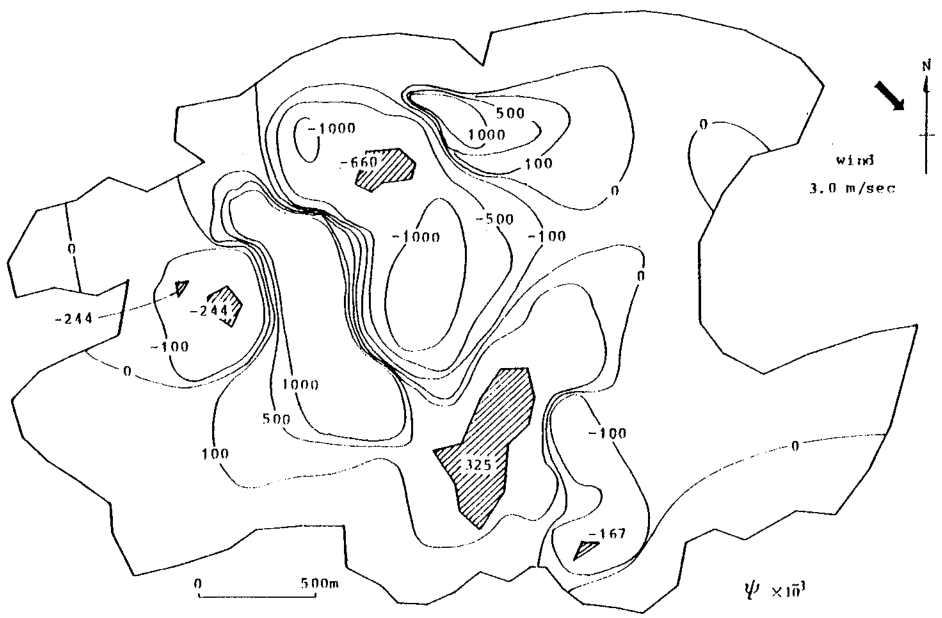

Fig. 4 Calculated stream function ψ.

islands in the lake and of mountains around the lake.

Fig. 4 shows the distribution of the stream function calculated under the above mentioned condition, from which the depthwise averages of the component velocities $\bar{u}$ and $\bar{v}$ can be calculated, using eq. (9). Then the vertical distribution of the velocities u and v can be calculated as a function of z.

Figs. 5, 6, 7 and 8 show the direction and magnitude of velocities at the water surface, and at the depths of 1m, 2m and 3m, respectively. The direction of the surface flow shown in **Fig. 5** is almost the same as the wind direction and the maximum surface velocity is about 6cm/sec ($0.02\,U_z$). The velocities at the 1m depth shown in **Fig. 6** are very small and their direction varies from place to place, and the direction of the flow at the 2m and 3m depths shown in **Fig. 7** and **Fig.** 8 is almost against the wind direction.

A typical vertical distribution of the velocity is shown in **Fig. 9**. The flow direction changes from the wind direction at the surface to the reverse of this direction near the bottom in a clockwise manner, which is the influence of the Coriolis parameter and is called the Ekman spiral.

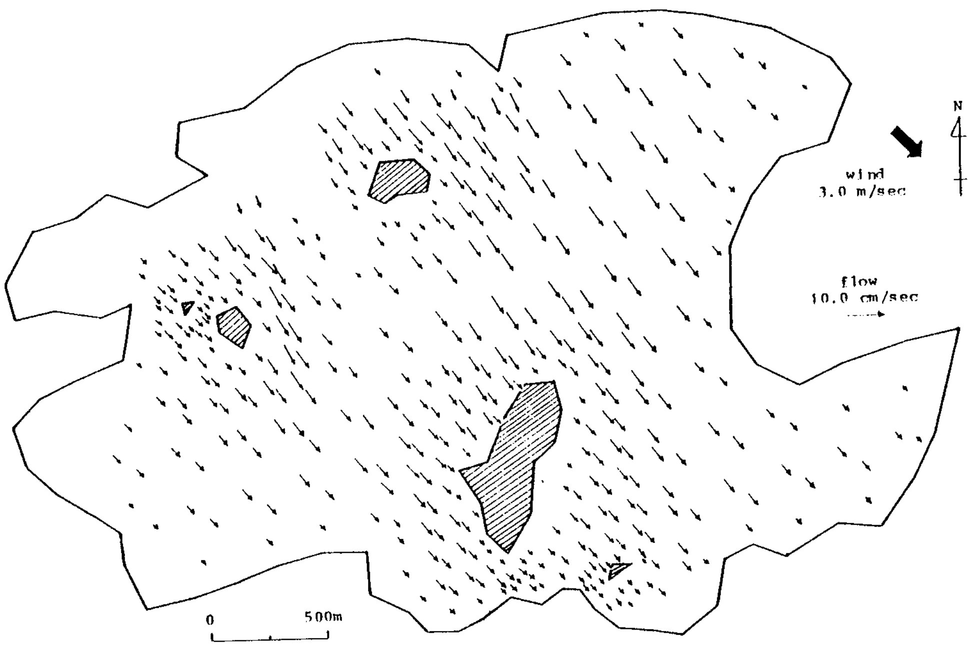

Fig. 5 Flow velocities at water surface.

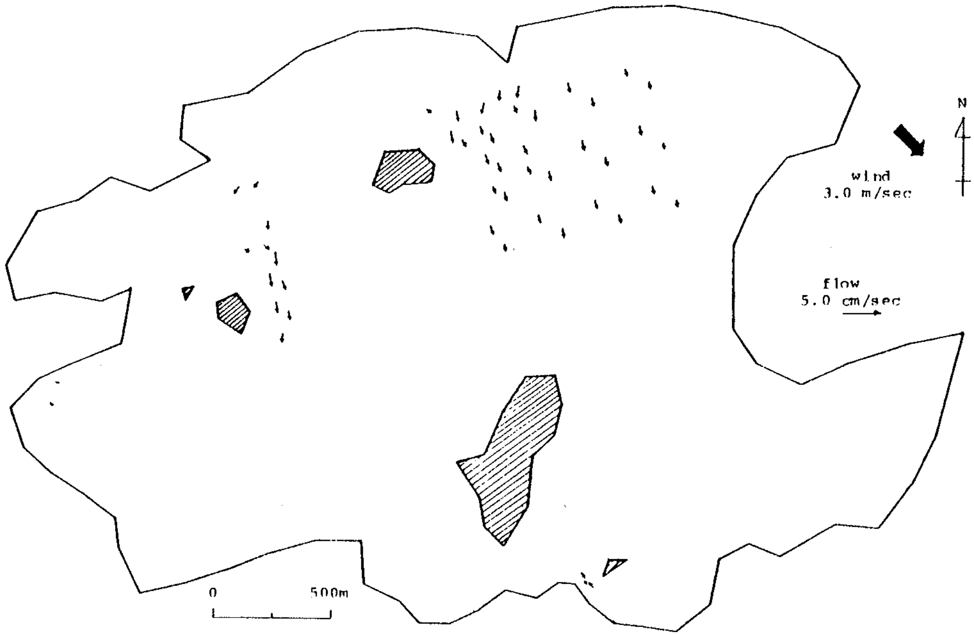

Fig. 6 Flow velocities at the depth of lm under the surface.

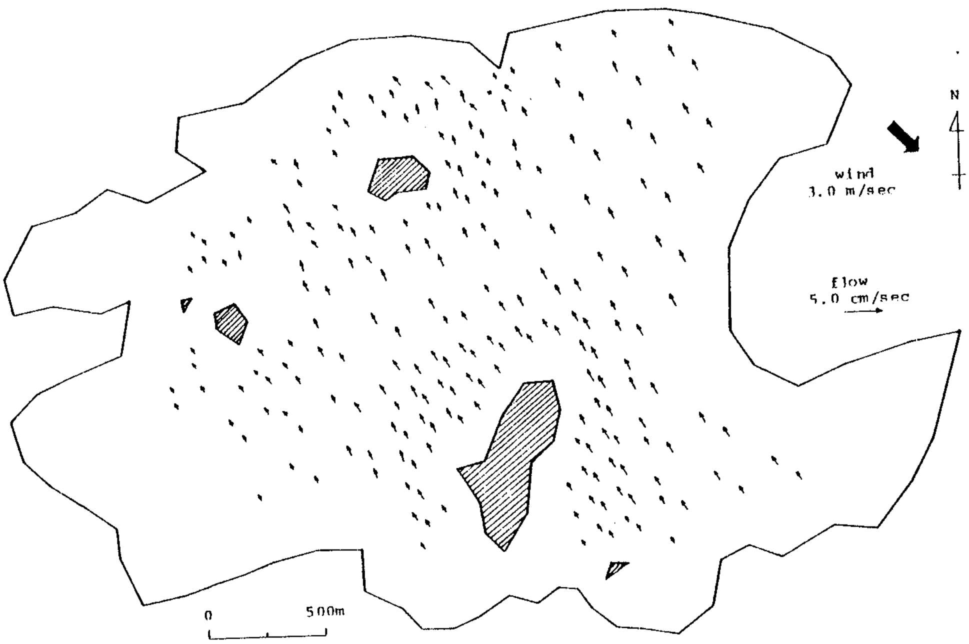

Fig. 7 Flow velocities at the depth of 2m under the surface.

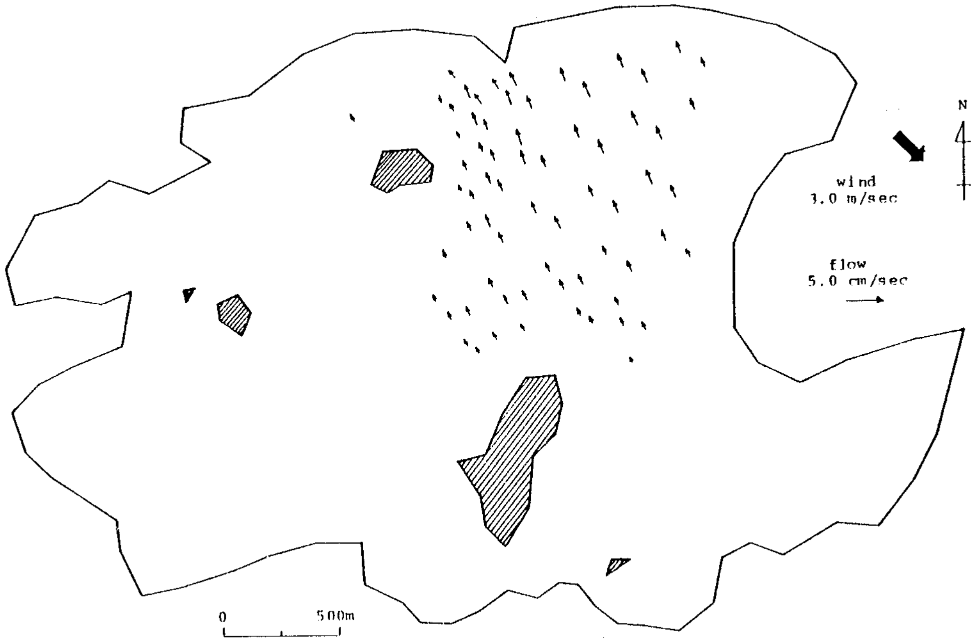

Fig. 8 Flow velocities at the depth of 3m under the surface.

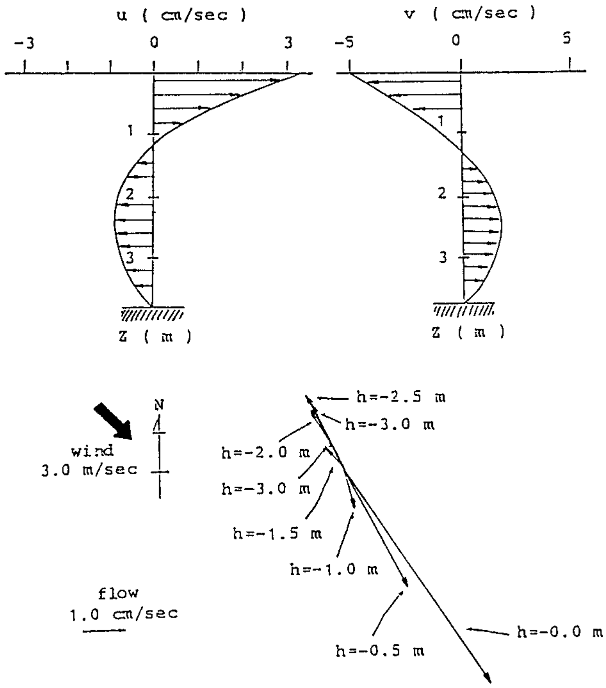

Fig. 9 vertical velocity distribution.

4. Observation

The circulation in Lake Koyamaike was observed on 14th and 27th January 1983, following floats by a pair of transits set at proper positions on the shore line. The float as shown in **Fig.**

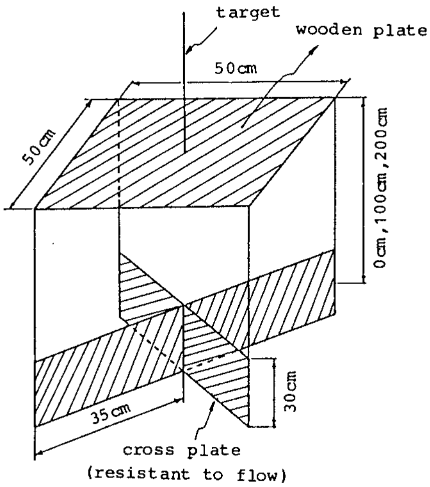

Fig. 10 Float for the observation.

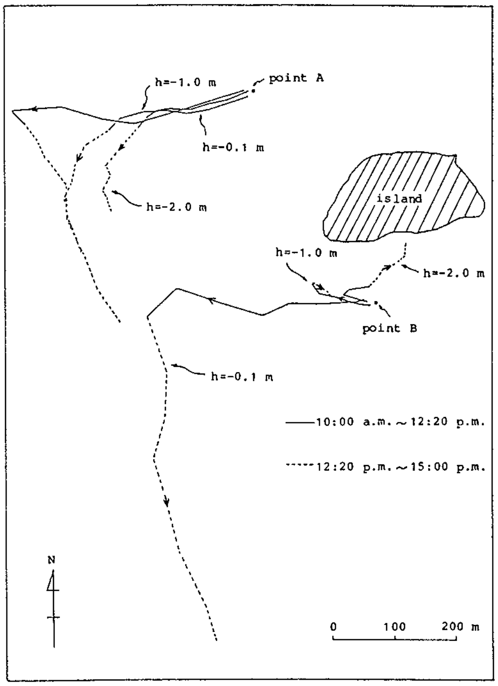

Fig. 11 Example of the float trails.

10 has two plates which cross each other and resist the flow. By adjusting the position of the plates, flow velocities at any depth under the water surface can be observed.

On 27th, an east wind of less than 1m/sec blew in the morning, but in the afternoon a north-west wind of about 3m/sec blew almost constantly. **Fig. 11** shows examples of the trails of the floats, in which solid lines indicate the flow in the morning and broken lines indicate the flow in the afternoon. **Fig. 12** shows the observed flow velocities compared with the simulated velocities mentioned in the previous section. The agreement between simulated and observed data is fairly good as a whole. But the wind condition for the simulation can not necessarily be the same as the condition of the field wind, because the wind in the field is not steady and sometimes changes its direction as well as its magnitude, and at relatively smaller velocities, agreement between the simulated and observed data is not so good.

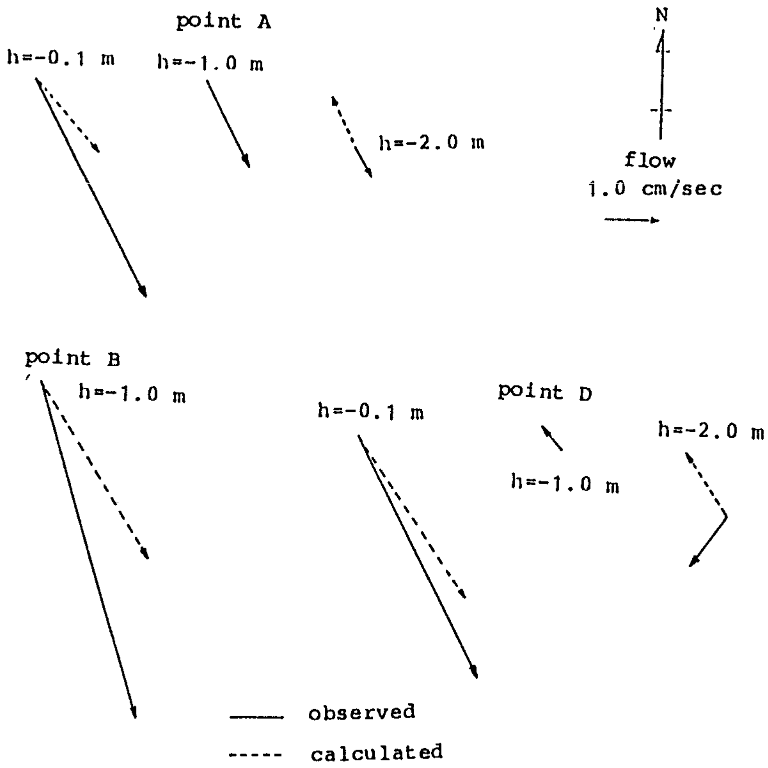

Fig. 12 Comparison between simulated and observed volocities.

5. Conclusion

The simulation method of the circulation in lakes proposed by Liggett et al., using the finite element method, was applied to Lake Koyamaike which has several islands, modifying the distribution of the wind velocities at the water surface. Also the characteristics of the circulation in the lake are presented, which may be useful for the future analysis of the transport phenomena of the sludge settled at the bottom.

The simulated circulation is proved to coincide with the data to some extent, but more detailed observation and simulation of the non-steady condition remain for future studies.

References

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Appendix I Notations

A, B, C = constant function of x, y and s in eq. (10) ;
a_1, b_1 = constants ;
C_f = coefficient of wind friction ;
D = typical vertical dimension used to normalize depth ;
d = Ekman depth of friction influence ;
f = Coriolis parameter ;
g = acceleration of gravity ;
h = water depth ;
L = typical horizontal dimension used to normalize width and length ;
p = local pressure ;
s = Laplace transform parameter ;
t = time ;
$\tilde{u}$, $\tilde{v}$ = average transformed velocity components in x and y directions, respectively ;
U_z = wind velocity at 10m above the water surface ;
x, y, z = Cartesian coordinates with x and y in a horizontal plane and z position upward and zero at the surface ;
η = eddy viscosity ;
ρ = fluid density ;
ρ_a = air density ;
τ_{wind} = surface wind stress ;
τ_x, τ_y = surface wind stress in x and y directions, respectively ;
ψ = stream function ;
$\sim$ = transformed variable ;
$_o$ = initial value ;
$*$ = nondimensional variable.

Appendix II Definitions

In this Appendix, all the variables except f, D and η in the definition of $m=D\sqrt{f/2\eta}$ are nondimensional and suffix $*$ is omitted.

$$A=\frac{1}{q_1}\left(\frac{\partial q_1}{\partial y}-\frac{\partial q_2}{\partial x}\right)$$

$$B=\frac{1}{q_1}\left(\frac{\partial q_2}{\partial y}-\frac{\partial q_1}{\partial x}\right)$$

$$C=\frac{1}{q_1}\left\{\frac{\partial}{\partial y}(q_4\tilde{\Gamma}'-q_3\tilde{\Delta}'+q_5)+\frac{\partial}{\partial x}(q_3\tilde{\Gamma}'+q_4\tilde{\Delta}'+q_6)\right\}$$

$$q_1=\frac{h_1}{h(h_1^2+h_2^2)}$$

$$q_2=\frac{h_2}{h(h_1^2+h_2^2)}$$

$$q_3=\frac{h_1h_3+h_2h_4}{h_1^2+h_4^2}$$

$$q_4=\frac{h_3h_4-h_1h_2}{h_1^2+h_4^2}$$

$$q_5=\frac{h_4h_6-h_1h_5}{h_1^2+h_4^2}$$

$$q_6=\frac{h_1h_6+h_4h_5}{h_1^2+h_4^2}$$

$$h_1=\frac{1}{ah(s^2+1)}\{-sh\alpha+(\gamma+\varepsilon s)(\beta+\kappa)+(-\gamma s+\varepsilon)(\delta+\lambda)\}$$

$$h_2=\frac{1}{\alpha h}\left\{\frac{\alpha}{M^2+N^2}\left(\frac{2MN}{M^2+N^2}+N\beta-M\delta\right)-(\gamma\beta+\varepsilon\delta)(\beta+\kappa)-(\beta\varepsilon-\delta\gamma)(\delta+\lambda)\right\}$$

$$h_3=\frac{1}{\alpha h}\left\{\frac{\alpha}{M^2+N^2}\left(\frac{M^2-N^2}{M^2+N^2}+M\beta+N\delta\right)+(\beta\varepsilon-\delta\gamma)(\beta+\kappa)-(\beta\gamma+\delta\varepsilon)(\delta+\lambda)\right\}$$

$$h_4=\frac{1}{\alpha h(s^2+1)}\left\{-\alpha h+(\varepsilon-\gamma s)(\beta+\kappa)-(\gamma+\varepsilon s)(\delta+\lambda)\right\}$$

$$h_5=\frac{1}{\alpha h(s^2+1)}\left\{(\beta+\kappa)\left(\varepsilon\frac{\partial p_0}{\partial x}-2\gamma\frac{\partial p_0}{\partial y}\right)-(\delta+\lambda)\left(\gamma\frac{\partial p_0}{\partial y}+2\varepsilon\frac{\partial p_0}{\partial x}\right)-\alpha h\left(s\bar{u}_0+2\bar{v}_0\frac{\partial p_0}{\partial x}\right)\right\}$$

$$h_6=\frac{1}{\alpha h(s^2+1)}\left\{(\beta+\kappa)\left(\gamma\frac{\partial p_0}{\partial x}+2\varepsilon\frac{\partial p_0}{\partial y}\right)+(\delta+\lambda)\left(\varepsilon\frac{\partial p_0}{\partial x}-2\gamma\frac{\partial p_0}{\partial y}\right)+\alpha h\left(-3\bar{u}_0+3s\bar{v}_0-2\frac{\partial p_0}{\partial y}\right)\right\}$$

$$\tilde{\Delta}'=\tilde{\Delta}-\frac{1}{s^2+1}\{s\Delta_0+2\Gamma_0\}$$

$$\tilde{\Gamma}'=\tilde{\Gamma}-\frac{1}{s^2+1}\{-3\Delta_0+3s\Gamma_0\}$$

$$\alpha=\cos^2 Nh(e^{-Mh}+e^{Mh})^2+\sin^2 Nh(e^{-Mh}-e^{Mh})^2$$

$$\beta=\frac{e^{-Mh}}{M^2+N^2}(N\sin Nh-M\cos Nh)$$

$$\gamma=\sin Nh(e^{-Mh}-e^{Mh})$$

$$\delta=\frac{e^{-Mh}}{M^2+N^2}(M\sin Nh+N\cos Nh)$$

$$\varepsilon=\cos Nh(e^{-Mh}+N\cos Nh)$$

$$\kappa=\frac{e^{Mh}}{M^2+N^2}(N\sin Nh+M\cos Nh)$$

$$\lambda=\frac{e^{Mh}}{M^2+N^2}(M\sin Nh-N\cos Nh)$$

$$M=\sqrt{2}\,m\sqrt{R}\,\cos\frac{\Phi}{2}$$

$$N=\sqrt{2}\,m\sqrt{R}\,\sin\frac{\Phi}{2}$$

$$R=\sqrt{1+s^2}$$

$$\Phi=\tan^{-1}\frac{1}{s}$$

$$m=D\sqrt{f/2\eta}$$