A Microcomputer Based Cost Allocation Gaming Analysis

by

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With a focus placed on cost allocation, a new approach has been presented in this paper. This new approach which makes full use of a microcomputer linked with a color monitor is intended to play a role of spanning over a gap between the normative type of approach as most of the conventional methods developed for cost allocation, and the empirical type of approach as known by the name of gaming. Though it is empirical and learning-oriented in the way it operates, this new approach may be strictly differenitated from gaming in common terms, owing to the former's special structure characterized by the built-in basic normalities. So the approach was designed to serve for both education and problem-finding.

Close study of the results of experiments has clearly demonstrated that it serves for the intended purposes. The power of the introduced microcomputer system has been discussed in detail. Suggestion is made of the needed further improvements of the presented approach.

1. Introduction

In the field of water resources management, there have been mounting concerns about how to reconcile conflicting interests among the different parties involved. Among a variety of conflict problems is the well known problem: how to split the total costs of a joint project among different users. This problem, which is generally called "cost allocation" is the major concern of this paper.

The water resources field has extensive literature on this theme. Many approaches have been proposed, tested and modified therein, and some of them appear to have gained extensive publicity and application in this field. The most conspicuous among them is the Separable Cost Remaining Benefit (SCRB) Method. This method, whose origin dates back to the early 1950's when a subcommittee of the Federal Interagency River Basin Committee recommended the SCRB, has been further developed in other countries to constitute the legal basis of present cost allocation procedures. In Japan it is prescribed by law that the allocation of costs should principally be performed by applying the SCRB-based procedure.

Though it is so widely applied, both theoretical and empirical studies have shown that

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SCRB has some crucial drawbacks and inconsistencies. Among them is the criticism that the conventional method including SCRB fail to handle the bargaining feature of cost allocation has provoked the development of a new approach in the water resources field. This approach owes its theoretical basis to what has been developed as the theory of cooperative games. It was not quite recently, however, that a systematic assessment was made of the implications and applicability of a cluster of game theoretic methods for the cost allocation in water resources management. From the viewpoint of equity and fair and common sense, Young, Okada, and Hashimoto[1),2)] have identified a set of basic principles that ought to be embodied in cost allocation, have then proceeded to a systematic check of both conventional and game theoretic methods against the basic principles. They concluded that the conventional methods including SCRB and some game theoretic methods fail to satisfy some of the basic principles and only a couple of lesser known methods from game theory; i. e. the Weak Nucleolus (WN) and the Proportional Nucleolus (PN), proved to be more appropriate. These points were illustrated by their application to a cost allocation problem among a group of Swedish municipalities developing a joint municipal water supply.

The development of the above study has motivated another type of approach. Ståhl[7)] has implemented an empirical approach called "gaming" to the Swedish cost allocation problem. He claimed that any allocation method based on preselected norms may not be accepted by participants. His approach was characterized by his position that the participants ought to be given as much free hand as possible in their bargaining with the others to find a final compromise. Invite players to the same table and let them play with the others, given a set of cost data on "going alone" and "going together". This was his idea.

Okada[3)] pointed out that the extent to which a cost allocation method has application may largely depend on the level or scope in which cost allocation is discussed and so there cannot be only one allocation method but rather many. He claimed that if a cost allocation enters in the project implementation phase as is commonly the case it becomes no more than a financial analysis and so demands a normative approach. Admittedly, there is another extreme situation in which empirical approach finds application. Suppose there has not yet been any established cost allocation procedure whatsoever and one desires to pick up those rules or norms which patternize what may turn out to be a normative procedure in the future. Ståhl's approach may be justified for this type of extreme situation. In practice, however, it appears more natural to assume that even when no procedure has yet been determined some minimum set of agreements or norms should be a priori set to base the negotiation game among them to find what may finally be developed into their cost allocation procedure. It is in this very sense that Okada[3)] has developed a "prescriptive-empirical approach" to cost allocation which intends to go halfway between the normative end and empirical end. There certainly are natural situations which demand this type of approach. The situations may include (i) when some or all of participants fail to understand the implications and validity of a normative method such as SCRB or a game theoretic method represented by WN and PN; (ii) when some or all are reluctant to accept the set of norms as it is although they may allow some basic ones to be retained; and (iii) with a set of norms proposed by the project manager or some of the participants, they desire to obtain a deeper understanding of what is implied by the application of these norms to cost allocation or they may even intend to add to the original set of norms. Very likely the situations may be compounded. Okada[3] has noted that a prescriptive-empirical approach to cost allocation which deals with such a situation ought to serve for both educational and problem finding purposes By education is meant an intention to get those ignorant acknowledge some normative wisdom and principles. Problem finding underlies a position which allows for latitude to individual experience, change and trial and error.

This study extends on his former study in the following points:

(1) Our new approach designs to incorporate a microcomputer in the procedure of cost allocation as an aid of supplying participants with information and explanations for the ongoing cost allocation gaming.

(2) The information is all visualized and colored to appear on the screen of a color display unit linked with the microcomputer.

(3) The rational for employing a microcomputer (and not a large computer) is owing to (i) economy and (ii) easiness and candiness with which to gain an access to it and to develop interactive dialogues with it. This is increasingly true as conspicously high speed of innovation in the microcomputer industry progresses year by year.

(4) After conducting experiments a number of times with participants seated before the computer, we will closely analyze the results from both a macroscopic and microscopic viewpoints. This will bring on to a systematic check of the applicability and limits of the proposed approach.

2. Design of Gaming

2. 1 Problem identified

Let us assume that three cities now contemplate to undertake a joint water supply project. Their primary concern is with how to allocate the tatal costs. So we have three players and not more than that. We will limit the number of players to three because (i) three players are the minimum condition for a coalition to be formed; (ii) the displaying of information in more than three dimensions entails technical difficulties; and (iii) a three-player game is considered the prototype of a coalition game. One may be allowed to go alone which would cost him what is termed as an individual cost of attaining the goal; or he may contemplate to go together with one of the rest to form a coalition against the last one who is forced to go alone. The cost of so doing is called a coalition cost or a joint cost. The datum on all of these costs to be estimated in advance is given in **Table 1**.

2. 2 Microcomputer system implemented

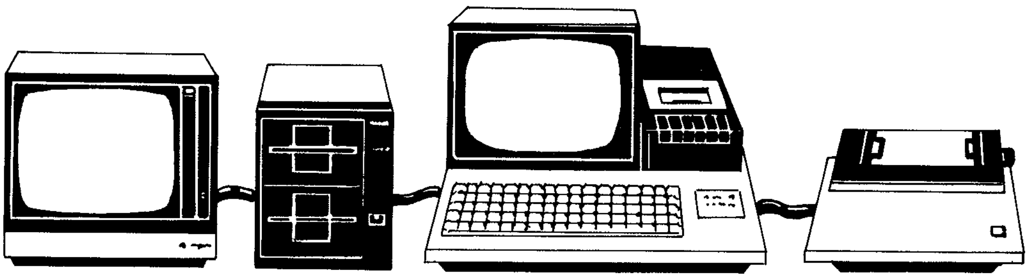
With costs and functions taken into account a choice has been made to implement the Sharp MZ-80 K2 microcomputer system which is composed of a " green computer " (main module), a dual floppy disk, a dot printer, a color monitor and interface units to link them together (see **Fig. 1**). The entire system costs some 1. 3 million yen or 5,400 US $.

INDIVIDUAL	COALITION	GRAND COALITION
C(A) = 6.5	C(AB)=10.3	
C(B) = 4.2	C(AC)= 8.0	C(ABC)=10.6
C(C) = 1.5	C(BC)= 5.3	

(UNIT : 10^8 yen)

Table 1 Input Cost Datum

Fig. 1 Microcomputer System Illustrated



2. 3 Minimum norms built in

Depending on whether players go alone or together, basic norms which have been reduced to minimum requirement are to be introduced in our approach. One is the self-evidence balance condition that a total of costs assigned to each be equal to the entire costs of the grand coalition project to be participated by all three cities. If no coalition is formed, the remaining condition is the principle of individual rationality which dictates that none of the participants be worse off by participating the grand project. Extension of this principle is made to the case in which a coalition is contemplated by two of the three; that is, the principle of group rationality which prescribes that a group contemplating to form a coalition not be worse off by participating in the grand coalition.

To formulate the above conditions in mathematical terms:

Self-evidence Condition:

$$X_A+X_B+X_C=C(ABC) \quad \cdots\cdots(1)$$

Individual Rationality:

$$X_A\leqq C(A);\ X_B\leqq C(B);\ X_C\leqq C(C). \quad \cdots\cdots(2)$$

Group Rationality:

$$X_A+X_B\leqq C(AB)$$

$$X_B+X_C\leqq C(BC) \quad \cdots\cdots(3)$$

$$X_A+X_C\leqq C(AC)$$

In the above X_i denotes the cost to be allocated to city i (i being A, B, or C) and $C(i)$ or C (S) represents the costs of the participant S as specified by the symbol parenthesized (S

being *AB, BC* or *AC*).

It is noted that the collection of the above three conditions gives the concept of core, a well known concept from the cooperative game theory as the basis of fairness and equity in bargaining and negotiation. Since it is assumed that "going alone" and "going together" are mutually exclusive in our cost allocation gaming and so only one of the two conditions, individual or group rationality is set to hold, there is no guarantee for a compromise solution to always satisfy core.

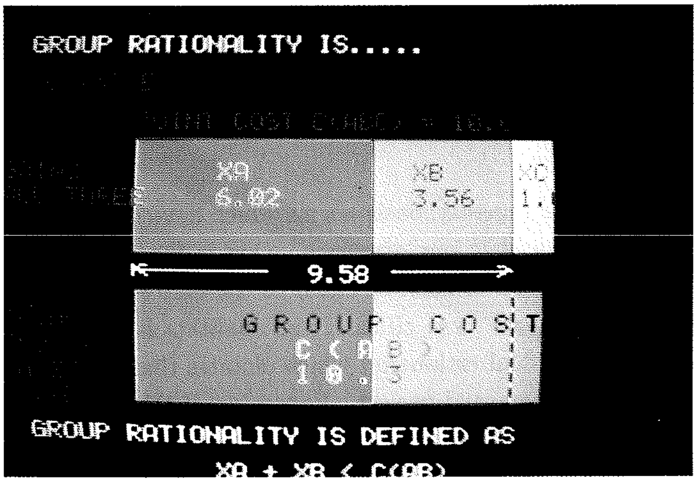


Photo 1 Pre-Gaming Guidance Information (1)

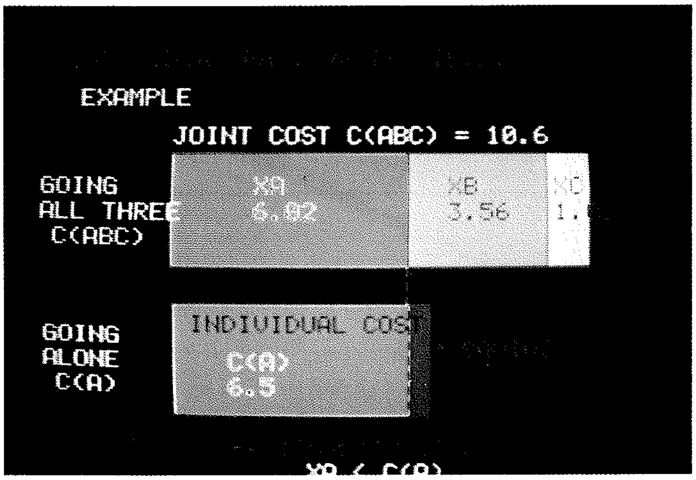


Photo 2 Pre-Gaming Guidance Information (2)

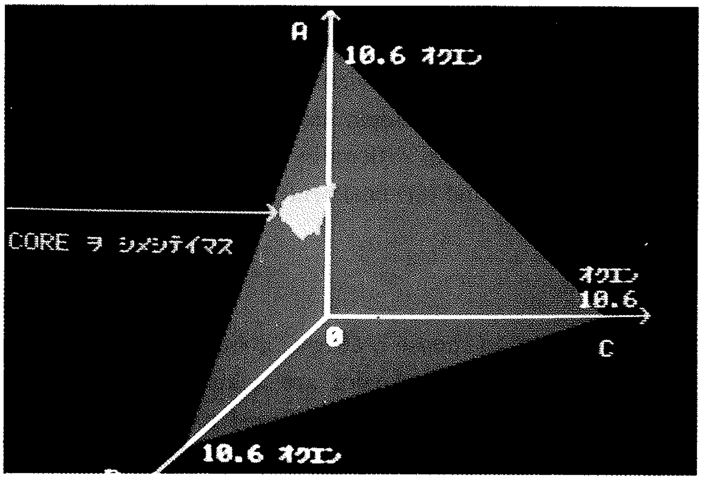


Photo 3 Pre-Gaming Guidance Information (3)

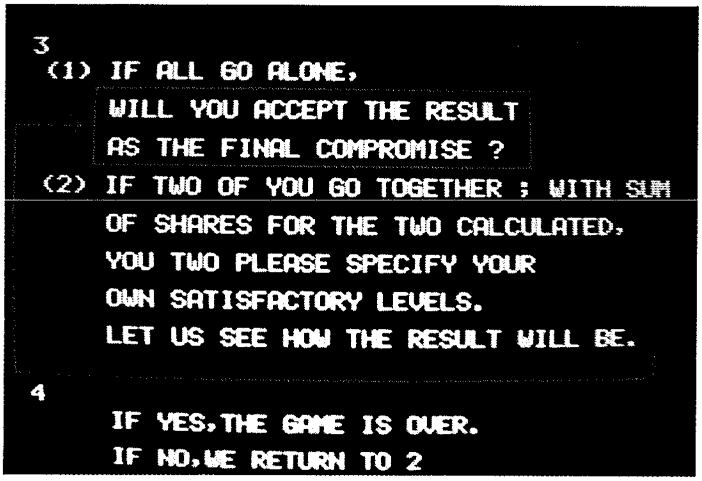


Photo 4 Pre-Gaming Guidance Information (4)

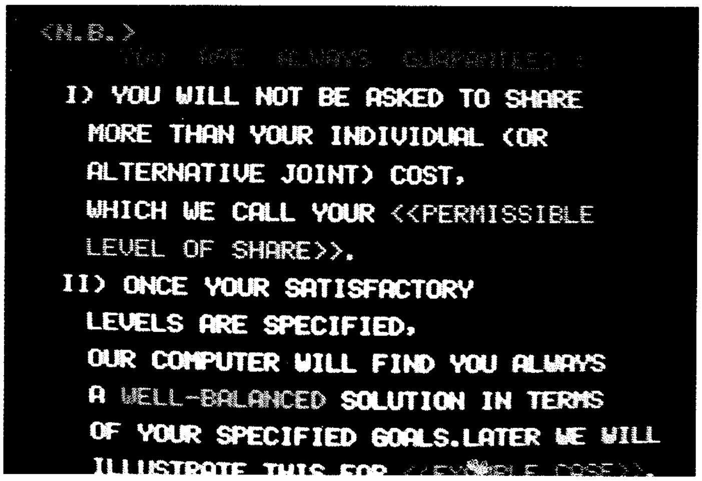


Photo 5 Pre-Gaming Guidance Information (5)

2.4 Pre-gaming guidance

By inviting three players to the game as the representatives of the three cities, we begin with supplying players with some guiding information which includes (i) cost data and possible coalition patterns; (ii) description of some minimum norms to base the game, i. e. self-evident condition, individual rationality and group rationality with the concept of core also illustrated for reference (see **Photos 1**to **3**); and (iii) basic rules and procedures for the gaming (see **Photos 4** and **5**).

2.5 Gaming

Gaming starts by asking each of the players to choose between "going alone" and "going together". Suppose all chose to go alone. Then players are asked to specify their "satisfactory level" for their share of the costs. Since it is designed to keep them from knowing what the others aspire as their satisfactory levels, they are asked to report to the gaming operator by ballot.

With the satisfactory levels thus fed in, the computer immediately tells them about what the automatically reconciled solution is. If every player finds it acceptable, which is rather unlikely in a very early stage of the gaming, we terminate the gaming and this solution is taken as their final compromise solution. Otherwise we go on with the same procedure until all agree to finalize the gaming.

2.6 Theoretical basis and its formulation

Once satisfactory levels are specified by either individual players or a group of players forming a coalition, the problem of finding a (provisional) comromise solution may easily be formulated as a multiobjective programming problem.

If no coalition is formed, the problem is written as:

$$\text{Minimize } X_A \quad \cdots\cdots (4)$$

$$\text{Minimize } X_B \quad \cdots\cdots (5)$$

$$\text{Minimize } X_C \quad \cdots\cdots (6)$$

subject to

$$X_A \leqq C(A);\ X_B \leqq C(B);\ X_C \leqq C(C) \quad \cdots\cdots (7)$$

$$X_A + X_B + X_C = C(ABC) \quad \cdots\cdots (8)$$

where inequality constraints come from individual rationality with X_A, X_B and X_C and *C(A), C(B), C(C)* and *C(ABC)* as defined before.

If a coalition is formed, there are two levels for the players in the group to go through in reaching a (provisional) compromise. With a coalition formed by, say, A and B,the level-one problem is formulated as:

$$\text{Minimize } X_A + X_B \quad \cdots\cdots (9)$$

$$Minimize\ X_C \quad \cdots\cdots (10)$$

subject to

$$X_A + X_B \leqq C(AB) \quad \cdots\cdots (11)$$

$$X_C \leqq C(C) \quad \cdots\cdots (12)$$

$$X_A + X_B + X_C = C(ABC). \quad \cdots\cdots (13)$$

The first inequality condition of Equation (11) dictates that group rationality should hold for a coalition *(AB)*, whereas individual rationality needs to hold for an individual player C as expressed by the second inequality condition of Equation (12).

On finding a provisional compromise solution for a coalition *(AB)* and an individual player C, as explained later, the level-two problem is to determine how to split between them the costs $\hat{X}_{AB}$ as assigned collectively to players A and B on level one. This lower level problem is played by the two, A and B who formed a coalition in the upper one.
This is expressed as:

$$Minimize\ X_A \quad \cdots\cdots (14)$$
$$Minimize\ X_B \quad \cdots\cdots (15)$$

subject to

$$X_A \leqq C(A);\ X_B \leqq C(B) \quad \cdots\cdots (16)$$

$$X_A + X_B = \hat{X}_{AB} \quad \cdots\cdots (17)$$

Again, the inequality conditions of Equation (16) are the expression of individual rationality to hold for A and B.

2. 7 Compromise finding algorithm

As is clear from the formulations above, the problem has been converted into a multi-objective programming problem, to which a variety of techniques have been so far developed to locate a most acceptable solution (or satisficing solution), not an optimal solution from a single objective viewpoint. Among many candidates has been singled out a technique of the goal programming with an explicit assumption of L-shaped utility function. This may easily be advocated by all the players who may be more likely to compromise when they find everyone's goal better balanced than otherwise.

By "well-balanced" we mean that the extent to which the achievement of one's objective is remote from his satisfactory level needs to be as close as possible to the extent to which the achievement of the other's objective is remote from his (the other's) satisfactory level. It is noted that another level called a permissible level is defined as the level the corresponding goal ought to reach at least. We take either individual or group rationality to stand for the permissible level for each player or a group, respectively.

The underlying idea as graphed in **Fig. 2** is that the well-balanced solution should lie, if possible, precisely on the line connecting between the two points, one representing each one's satisfactory and permissible levels; otherwise it ought to be as close as possible to the line. This line is termed as a "goal vector." If one cannot know which situation is to occur, linear programming needs to be made use of to calculate what is regarded as a well-balanced solution. Since one may easily prove that only the former situation takes place for our three dimensional problem as is clear from **Fig. 3,** the solution is analytically identified with the point which is the intersection of the goal vector and the plane for the self-evident total-cost balance condition.

In consequence a well-balanced solution which we conceive as a provisional compromise solution is given by the following formuli:

If no coalition is formed,

$$\hat{X}_i = \bar{g}_i + (\lambda_i / \Sigma \lambda_i) \times \{C(ABC) - \sum_i \bar{g}_i\} \quad \text{for } i = A,\ B \text{ and } C, \qquad \cdots\cdots(18)$$

where $\lambda_i = C(i) - \bar{g}_i$ with $C(i)$ which is player i's individual cost taken as his permissible level and $\bar{g}_i$ represents his satisfactory level.

If a coalition is formed by A and B just by way of explanation, the level-one allocation is given as:

$$\hat{X}_{AB} = \bar{g}_{AB} + \{\lambda_{AB} / (\lambda_{AB} + \lambda_C)\} \times \{C(ABC) - (\bar{g}_{AB} + \bar{g}_C)\} \qquad \cdots\cdots(19)$$

for a coalition *(AB)*

$$\hat{X}_C = \bar{g}_C + \{\lambda_C / (\lambda_{AB} + \lambda_C)\} \times \{C(ABC) - (\bar{g}_{AB} + \bar{g}_C)\} \qquad \cdots\cdots(20)$$

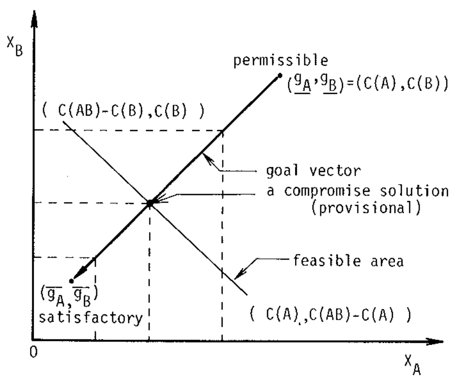


Fig. 2 Well-Balanced Solution on Two-Goal Space

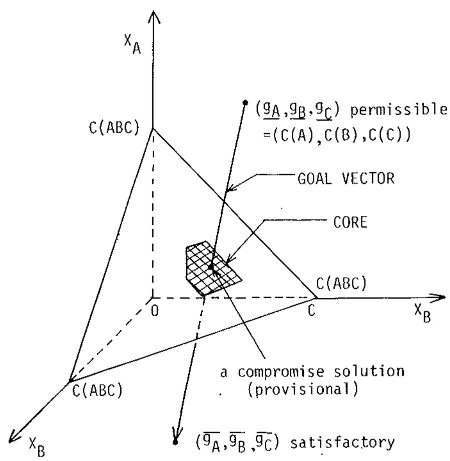


Fig. 3 Compromise Solution Located on ThreeDimensional Space

for the remaining individual C.

With $\hat{X}_{AB}$ given as such, the level-two allocation reads as:

$$\hat{X}_i = \bar{g}_i + (\lambda_i / \sum_i \lambda_i) \times \{\hat{X}_{AB} - \sum_i g_i\} \qquad \cdots\cdots (21)$$

for $i = A$ and B for the example of A and B forming a coalition.

2. 7 Man-machine interactive dialogues

By programming these formuli on the microcomputer and on feeding it with the datum on the satisfactory levels specified by the players, the solution is instantaneously calculated to appear on the screen. The players can also have access to some other visual information as illustrated by **Photos 6 and 7.** The picture shown in the former photo informs the players of where the solution is located and whether it is in or out of core. If players want to keep track of the series of their past provisional compromise solutions which they have so far not

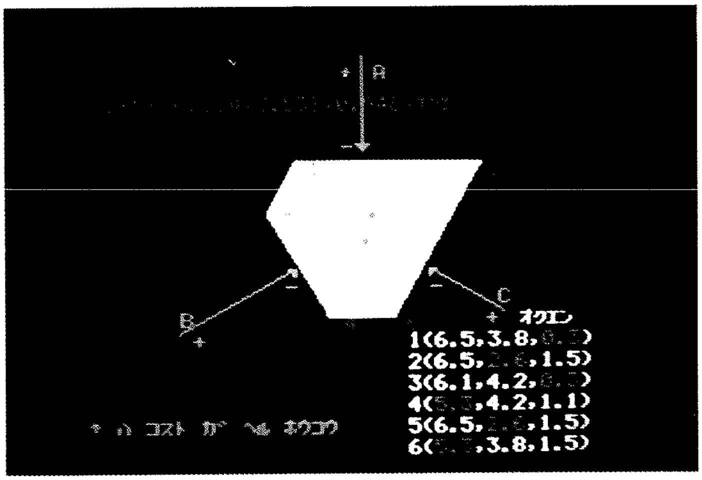


Photo 6 Provisional Results of Gaming on Display (1)

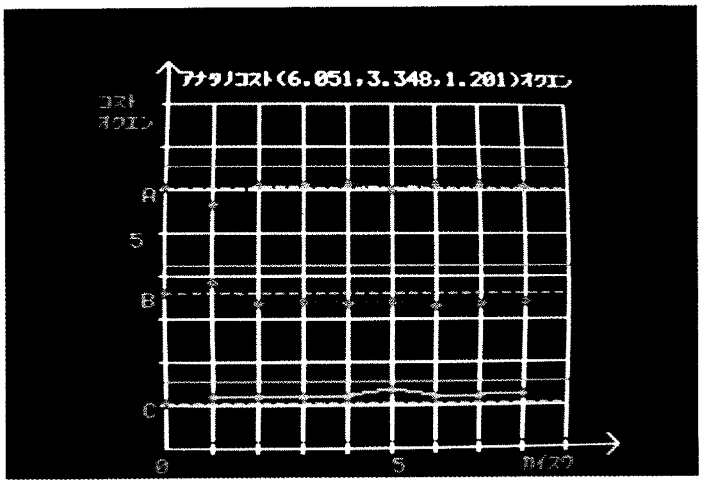


Photo 7 Provisional Results of Gaming on Display (2)

accepted in hope of finding a better one, the diagram as shown in the latter photo serves their purpose. The idea is to get the players seated before the computer and let them play with the others by developing interactive dialogues with the rest of the participants with the machine as the media of communication.

3. Results of Experiments

3.1 Design of experiments

By thus developing a microcomputer based information system for the cost allocation gaming, a total of 60 students were invited to the forum, thus producing the results of 20 cases. The students, undergraduate or graduate students, come from the Department of Civil engineering, Tottori University. Each time three students were asked to be seated before the computer with the author as the operator and referee for the gaming. They were allowed to play not more than eight rounds. The limit to eight rounds is grounded on the assumption that if three of all are allowed to try two courses of action, namely, going alone or going together with someone, the number of possible outcomes is $2^{8}=8$.

If players found still hard to compromise within an allowed number of eight rounds, they were asked to rate each of their former provisional compromise solutions. This rating by each player is reported only to the operator, who singles out one of the solutions that is rated "averagedly highest" by the participants. By "averagedly highest" is meant the solution for which the rating is averaged over the three players to rank highest.

In each round of the game the players were asked to fill out a questionnaire on the following items: (i) the reason for either accepting or not accepting the current provisional compromise solution, (ii) selection of courses of action, i. e. going alone or going together;

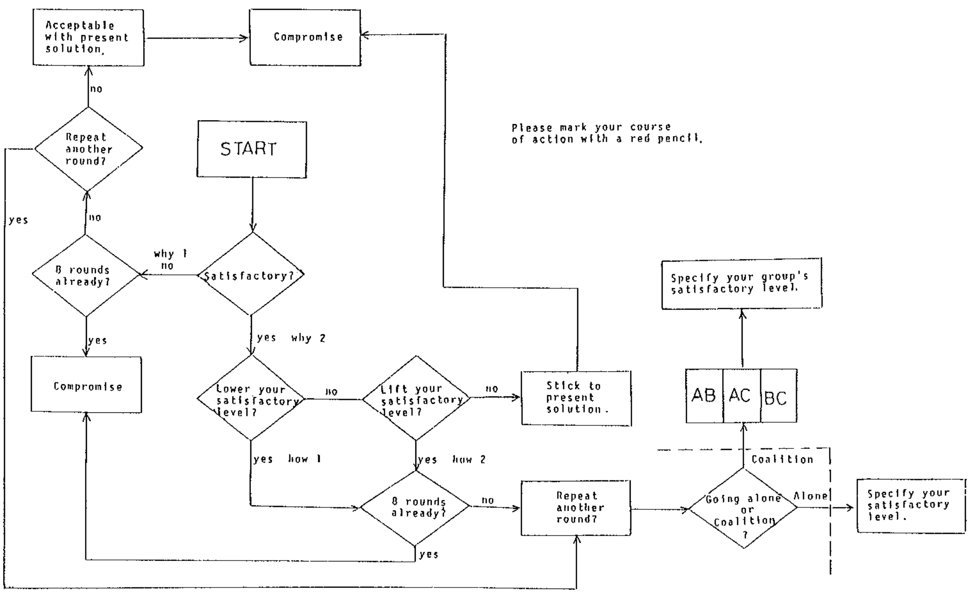


Fig. 4 Questionnaire

and the latter being the case, with whom?, and (iii) specification of one's satisfactory level (changeable in each round) (see **Fig. 4**).

When the game is over, they were questioned about (iv) the degree to which one's understanding has improved of the key built-in allocation normalities, and (v) the rating of one's preference among a given set of reference methods of cost allocation, that is, SCRB, Shapley Value, Nucleolus, Weak Nucleolus (WN) and Proportional Nucleolus (PN).

3. 2 Analysis of results

1)patternization of results

Table 2 lists the twenty cases of empirical results. For analytical convenience by dividing the range of values into three the final compromise values were classed into three categories, "high" denoted by H (unfovorable), "medium" denoted by M, and "low" denoted by L (favorable). **Table 3** and **Fig. 5** show the histograms of the final cmpromise values for the three players. Study of this table immediately indicates that:

(1) Players A and C outrank B in the number of those who finally accepted relatively high

Table 2 Empirical Results

Case	A	B	C
1	5.817	3.553	1.230
2	5.745	3.687	1.168
3	6.065	3.362	1.173
4	5.900	3.500	1.200
5	5.723	3.756	1.121
6	5.970	3.316	1.313
7	5.937	3.675	0.988
8	5.620	3.747	1.233
9	5.900	3.494	1.206
10	5.802	3.659	1.139
11	5.649	3.689	1.262
12	6.084	3.327	1.188
13	5.934	3.577	1.089
14	5.658	3.767	1.175
15	5.927	3.688	0.988
16	5.709	3.637	1.254
17	5.851	3.508	1.241
18	5.381	3.919	1.299
19	5.911	3.685	1.003
20	5.548	3.791	1.261

(UNIT: 10^8 Yen)

Table 3 Compromise Values Categorized

CASE	A	B	C
1	M	L	M
2	M	M	L
3	H	L	M
4	M	L	M
5	L	H	L
6	H	L	H
7	H	M	L
8	L	H	H
9	M	L	M
10	M	M	L
11	L	H	H
12	H	L	M
13	H	M	L
14	L	H	M
15	H	H	L
16	L	M	H
17	M	L	H
18	L	H	H
19	H	M	L
20	L	H	H

L: low value
M: medium
H: high

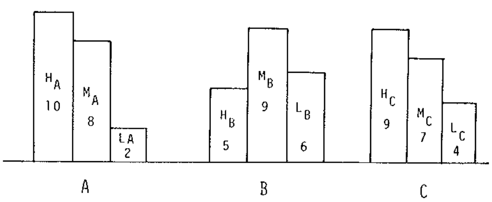


Fig. 5 Histograms of the Distribution of Compromise Solutions

a cost or relatively unfavorable an allocation.
(2) Comparatively, player B outranks players A and C in the number of those who fall in the middle class.
(3) As for players A and C the number of those who enjoy relatively low a cost is the smallest, whereas it is relatively large for player B.
(4) Collectively players A and C share a pattern of right-downward distribution, as against B which is patternized by its cone-shaped distribution. (4) To reinterprete, a microscopic feature of the results is characterized by seemingly favorable values for player B against A and C. This may be justified by the fact that player B is in the strongest position to claim a cost relatively low for him on the ground that the contribution of A and C to their joint venture and the grand project in increasing economic efficiency is nothing as compared to that of B, as is clearly structured in the cost input datum in **Table 1.** This means that the outlined dintribution pattern of compromise values is determined largely by the structure of the cost input datum. (5) As far as a set of compromise values for players A, B and C are concerned, the pattern which occurs most frequently has proved to be (H_A, L_B, M_C), indicating that A, B and C belong to class H, L and M, respectively. Again, this reassures the general trend of favorable players for B.

2) Comparative analysis

(1) For vehicle of comparison it might be of analytical interest to apply the above symbolic system to a set of the other cost allocation techniques. Then we get Nucleolus=(H_A, L_B, L_C), WN=(H_A, L_B, M_C), PN=(H_A, L_B, H_C), Shapley value=(H_A, M_B, L_C) and SCRB=(H_A, M_B, L_C). It follows from this that all but SCRB and Shapley Value share the general trend of favorable results for B as seen from the above gaming experiments. The pattern which is closest to our experimental results has proved to be that of PN.
(2) Another analytical interest is to examine whether initial solutions affect what they have finally agreed on. This underlies our suspicion that much of the game might be determined by just a single "pushing" player who can preempty the others by claiming exorbitantly favorable a value for himself at the outset of the gaming. A statistical test has been done to examine the significance of the differences between initial and final compromise solutions for each player. It can been shown that no statistical significance is gauged in the manner the former values deviate from the latter. So we may conclude that repetitive rounds of gaming helps players learn how they should act or react by forming a coalition where necessary, thus eventually converging onto a range of reasonable values, notwithstanding some minor exceptions of extreme values.

3) Comparative analysis

So far has been a macroscopic analysis of the results. We now turn our eyes to more microscopic features of the results. In another word we intend to take a closeup of the above question: how much player's bargainability counts in gaming. We base our analysis again on the questionnaire results.

We start with the definition of player's characteristics of "strong", "medium" and "weak". A "strong" bargainer is defined to be a person who wants to have another round, hoping to gain more even if he feels that the current solution is rather satisfactory for him; or who always finds any solution unacceptable as a final compromise; or who immediately starts feeling unsatisfactory with the current solution, if it turns out to be less favorable for him than the preceding one.

Likewise, by definition, a "weak" bargainer is a person who is ready to find some solution acceptable in relatively early a stage and rushes to compromise by giving up his present claim; or who rushes to accept the presest solution even if he finds it not necessarily so satisfactory. A "medium" bergainer is defined to be the rest, neither weak or strong.

In the above are underlined those paragraphs which we can get track of from the questionnaire. By applying the above definitions to our players, each player in a gaming has been marked with "*S*", "*M*" or "*W*", as shown in **Table 4.** Comparison of this table with **Table 3** which lists the compromise values categorized as *"H", "M"* and *"L"*, leads to **Fig. 6.** This figure shows the number of each player with a bargaining character categorized as *"S", "M"* or *"W"* against compromise values ranked as *"H", "M"* or *"L"*. A mere glance of this figure shows that irrespective of player *A, B* or *C,* it is highly likely that those "strong" bargainers tend to enjoy relatively "low" costs (favorable values), whereas those "weak" ones end up with relatively "high" costs (unfavorable values). This tendency may not be, however,

Table 4 Player's Characters Categorized

CASE	A	B	C
1	W	M	S
2	M	M	W
3	W	M	W
4	W	W	W
5	W	W	S
6	S	S	S
7	W	S	S
8	M	W	W
9	W	S	W
10	W	W	M
11	S	W	M
12	W	S	S
13	W	M	S
14	S	S	S
15	W	M	S
16	S	M	W
17	W	S	W
18	S	W	W
19	W	W	M
20	S	W	M

S:strong bargainer

M:medium

W:weak

C

Bargaining Character	Compromise value H	M	L	
S	4	3	1	8
M	2	0	2	4
W	1	3	4	8
	7	6	7	

B

Bargaining Character	Compromise value H	M	L	
S	4	1	1	6
M	2	3	1	6
W	1	2	5	8
	7	6	7	

A

Bargaining Character	Compromise value H	M	L	
S	5	0	1	6
M	1	1	0	2
W	1	5	6	12
	7	6	7	

Fig. 6 Comprorise Values vs. Player's Characteristics

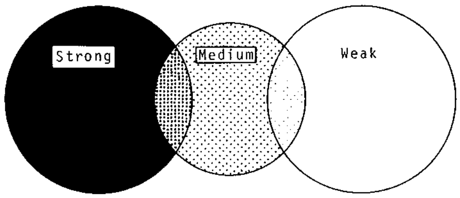


Fig. 7 **Bargainer's Characteristics Discriminated**

so clear for those who fall in the category of "medium". A statistical test has been done to examine this hypothesis: one's barganing character affects what he will finally gain.

The result partially supports this hypothesis with a significance level of five percent; that is, significance has been gauged between any two cases, one which was played by a "weak" *A* *(B* or *C)* and another played by a "strong" *A (B* or *C,* respectively), though no significance has been picked up between any two cases where one of the compared two players was found to be "medium". This fact is illustrated in **Fig. 7.**

To conclude we may fairly say that microscopically a cost allocation gaming so defined is subject to player's bargainability to a limited extent, if we compare a particular outcome with another.

4) Complementary analysis

By reference to the results of the post-gaming questionnaires, some complementary analysis has been conducted to find the following:

(1) 85 percent of those who had failed to understand the concept of core before they became involved in the gaming, admitted that the implication of the concept became clearer to them.

(2) 80 percent of those who finally gained the understanding of core found it a reasonable condition for cost allocation.

(3) According to the rating of five other alternative cost allocation methods, SCRB has been found to rank top, which is followed by Shapley Value, then by PN, then by WN and finally

by Nucleolus.
(4) The most popular criterion employed by participants for so rating was the easiness to understand and the least mathematical complexties which entail a given method.

It might not be overstated that this easiness to use and understand attracts practitioners and laymen. For this reason SCRB and Shapley Valye tend to be rated before the rest. This may well explain why SCRB still gains so much popularity for all flaws it entails.

It seems contradictory, however, that players rate both SCRB and Shapley Value (which do not necessarily satisfy core) higher than the other core-based techniques, even though they supported core as a reasonable condition for cost allocation. This is explained partly by the possibility that their understanding of core was not enough. Another reason may be that in our example both SCRB and Shapley Value always happened to satisfy core, which means that it seems very likely that either SCRB or Shapley Value could have violated core if we had used a slight different example.

It should be noted that all participants have felt that our cost allocation gaming is also appropriate in terms of easiness to use and understand. Many of them have agreed that this type of experimental technique could serve the purpose of educating people to become more familiarized with the problem of cost allocation, leading them up to the essential question of "what is fairness and equity?".

3.3 SCRB as part of gaming

Finally it might be of additional interest to refer to the fact that our prescriptive-empirical gaming approach offers an reinterpretation of SCRB, because the former comprises the latter. That is, it may easily be demonstrated that the SCRB based solution is no more than a special solution among a set of possible compromise solutions for our gaming. The point to be made is the assumption that each player who chose to go alone has agreed to take his own "separable cost" as a satisfactory level. One's "separable cost" is defined as the cost of the particular participant leaving the grand project or as the marginal cost of adding him to the list of participants as the last one.

This assumption which is called marginality principle may seem rather natural to players with common sense and reasoning, because otherwise he would be forced to leave the grand project, which in turn leaves him with no choice but to go alone, which would cost more than his separable cost. On substituting one's individual cost into his satisfactory level $\bar{g}_A$ in Equation (18), we get for player A

$$\hat{X}_A=\bar{g}_A+\lambda_A/(\lambda_A+\lambda_B+\lambda_C)\times\{C(ABC)-(\bar{g}_A+\bar{g}_B+\bar{g}_C)\} \quad \cdots\cdots (18)$$

$$=\bar{g}_A+(C(A)-\bar{g}_A)/\{C(A)-\bar{g}_A+C(B)-\bar{g}_B+C(C)-\bar{g}_C\}$$

$$\times\{C(ABC)-(\bar{g}_A+\bar{g}_B+\bar{g}_C)\}$$

$$=SC(A)+\{RB(A)/(RB(A)+RB(B)+RB(C))\}$$

$$\times\{C(ABC)-(SC(A)+SC(B)+SC(C))\} \quad \cdots\cdots (22)$$

which is precisely the allocation formula of SCRB with each one's individual costs being lesser than his benefits. In the above SC stands for one's separable cost and RB denotes one's remaining benefit which is defined to be the difference between one's individual costs and separable costs. Therefore it holds for $i=A$, B and C that:

$$RB(i)=C(i)-SC(i)=C(i)-\bar{g}_i \qquad \cdots\cdots (23)$$

It is clear that Equation (22) applies for any participant other than A.

The above fact indicates that our new approach works very well for the purpose of gaining further insight into SCRB and offers a game-theoretic reinterpretation of what is implied by the method. If participants so desire, they can take as their cost allocation the SCRB based solution for the gaming.

4. Conclusion

In this paper we have presented a prescriptive-empirical approach to cost allocation with an aid of a microcomputer based information system and have demonstrated how well our approach serves our purpose.

A point of departure from conventional approaches was the awareness of the need among practitioners and laymen for an advent of a new approach which could complement, not to say replace, any of the conventional methods or newly developed game-theoretic methods. The new direction was intended to be somewhere between normative approaches which include almost all conventional methods and empirical approaches known by the name of gaming. What destinguishes our prescriptive-empirical approach for an ordinary type of gaming was that the former is guided by the minimum normalities incorporated in the gaming procedure.

Some major findings may be summarized as follows:

(1) Despite minor differences among different cases, the outlined distribution pattern of compromise values is determined largely by the structure of the input cost datum, and not so much by the bargainability of players. This is very much owing to those basic norms incorporated in the gaming which guide much of the direction of the gaming. This explains why the presented approach is called a "prescriptive-empirical" approach, not simply an empirical approach or a gaming.

(2) From a microscopic point of view, however, this is not necessarily the case. One's bargaining power makes some difference. Therefore we may say that players are given limited free hand as long as they stay within the predetermined conditions incorporated in the gaming procedure.

(3) The microcomputer-aided approach has proved to be very effective and helpful in educating people who are not familiar with cost allocation. Very often people tend to disagree with a given approach simply because they fail to gain a full understanding of it. By so familiarizing them with the essence of cost allocation people will be more likely to accept it, as was precisely the case with our experiments. It also serves for the purpose of bringing

up the practitioners to a round table before the computer to let them express what they expect a method of cost allocation to be. Very likely they will come to learn that they have ended up with contradicting not only the others but themselves. A glance of the track of their outcomes which are colorfully visualized on the screen will readily tell them about this. It is in this very sense that our prescriptive-empirical approach is regarded as an education-oriented and problem-finding approach.

(4) By incorporating what has been agreed on through experimentation into the gaming procedure, we may expect to add to the normalities for guiding the game, thus eventually leading closer to a more normative type of methodology.

With all benefits of our approach, there seems to be much room for extension and improvement. A list of technical difficulties to overcome includes: (i) how to visualize more than three dimensional information on the screen of a color monitor linked with a microcomputer if more than three players are involved in the cost allocation; and (ii) how to speed up the processing and display of information, and how to overlay one image on another in order to make the presentation of information more attractive and effective.

We could certainly overcome them with a larger scale of computer but our major concern is how to make it on a microcomputer. A remarkable speed of advancement in the microcomputer industry seems to offer us a rather promising prospect.

Another concern of ours is to invite practitioners and managers experienced in the business of cost allocation to play the game by themselves. By accommodating their advice and criticism we may develop a more applicable approach in line with the approach suggested here.

A step forward has already been taken with some enocouraging fruits, which will be presented in our forthcoming paper.

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