Calculation of the Static Pressure Distribution around a Circular Cylinder with Tangential Blowing

by

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The circumferential static pressure distributions around a circular cylinder with tangential blowing were calculated by using the modified Parkinson-Jandali's method, extended to an asymmetrical flow. It was confirmed that the distributions calculated by the present method were well consistent with the experimental results except for the neighbourhood of the separation points.

1 Introduction

It is one of the very important and basic problems in practice to estimate an aerodynamic force acting on a bluff body immersed in a uniform flow. Differing from the case of a streamlined body, the flow past a bluff body is always accompanied with the flow separations and a broad wake region caused by its rear shape of the body. The theoretical calculation of the flow around a bluff body at the high Reynolds number has been mostly carried out based on the free streamline theorem for an inviscid fluid.

Though many flow models have been considered up to date, there are no theories which can predict or describe all of the characteristics of the flow around such a body because of the complexity of the wake dynamics, the viscous effect, and so on.

Nowadays the static pressure distribution and a drag coefficient can be calculated fairly well for a symmetrical flow by the free streamline theorem, though of course the separation points and the base pressure (assumed constant over the separated region) obtained empirically have to be prescribed. These calculations are made for the flows past symmetrical bodies with respect to the incident streamline, e. g., for those past a flat plate, a 90° wedge, a circular cylinder, an elliptical cylinder, and so on.[1),2)] As far as the authors know, however, there are few examples applying the free streamline theorem to the asymmetrical flow.

In this report the authors described the calculation method which was the extended Parkinson-Jandali's method[1)] and could be applied to the asymmetrical flow such as the flow past a circular cylinder with tangential blowing in a uniform flow. The calculated pressure

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distributions are compared with the experimental results. And it was confirmed that both distributions by the theory and experiment fairly well agreed with each other except for the neighbourhood of the separation points.

2 Main nomenclatures

C_p = pressure coefficient.
C_{pb} = base pressure coefficient.
C_μ = (momentum of the jet per unit span)/[(1/2)$\rho_\infty U_\infty^2 D$] .
D = diameter of the circular cylinder.
R_e = Reynolds number ($=U_\infty D/\nu$).
U_∞ = velocity of the uniform flow.
θ = angle measured clockwise from the leading edge of the cylinder.
θ_j = angular location of the slot.
θ_u & θ_l = angular locations of the separation points on the upper and lower surfaces of the cylinder, respectively.
ν = kinematic viscosity coefficient.

3 Theory

The calculation method of a static pressure distribution, described in this report, is based on the Parkinson-Jandali's method which is applicable to a two-dimensional, incompressible potential flow external to a symmetrical bluff body and its wake. Here, the Parkinson -Jandali's method is extended to an asymmetrical flow past a circular cylinder with tangential blowing.

Fig.1 shows basic and physical planes, that is, the T-plane and the t-plane, respectively. Any point in the T-plane, $T=X+iZ$ (i is imaginary unit), is transformed to the point, $t = x+iz$, in the t-plane by the transformation,

$$t=f(T)=e^{-i\gamma}\left(T-\cot\delta-\frac{1}{T-\cot\delta}\right) \qquad \cdots\cdots (1) ,$$

where γ and δ are the angles between the incident flow and the X-axis and between the stagnation point and the X-axis in the T-plane, respectively. Then, the complete circle, AS_1BS_2A, of which the center is at the origin is conformally mapped to a circular arc slit, $A'S_1'B'S_2'A'$, in the t-plane by Eq.(1). This circular arc slit mapped from the circle is not symmetrical but asymmetrical with respect to the x-axis. The stagnation points S_1 and S_2 in the T-plane are mapped to the separation points S_1' and S_2' in the t-plane, respectively and the stagnation streamlines at S_1 and S_2 are transformed to the tangential separation streamlines at S_1' and S_2'.

Now, consider the two-dimensional, incompressible, potential steady flow past a circular cylinder immersed in a uniform flow inclined by the angle γ with respect to the X-axis. The

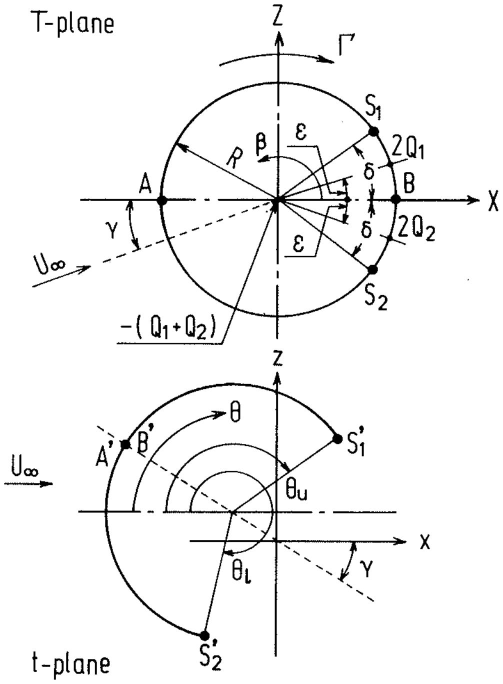

Fig.1 Basic and physical planes.

complex potential $W_1(T)$ of this flow is expressed as follows;

$$W_1(T)=U_\infty(Te^{-i\gamma}+\frac{R^2}{T}e^{i\gamma}) \quad \cdots\cdots (2) ,$$

where U_∞ and R are the velocity of the uniform flow and the radius of the circular cylinder, respectively. In addition to this, consider (1) two sources of different strength $2Q_1$ and $2Q_2$ at symmetrical angular locations ε and $-\varepsilon$ on the circumference of the circle, respectively, (2) a sink of strength (Q_1+Q_2) at the origin, and (3) a circulation of strength Γ around the circle (see Fig. 1).

Then the total complex potential $W(T)$ of the asymmetrical flow with a broad wake region in the T-plane is expressed as follows;

$$W(T)=U_\infty(Te^{-i\gamma}+\frac{R^2}{T}e^{i\gamma})+\frac{Q_1}{\pi}ln(T-Re^{i\varepsilon})+\frac{Q_2}{\pi}ln(T-Re^{-i\varepsilon})$$

$$-\frac{(Q_1+Q_2)}{2\pi}lnTe^{-i\gamma}+\frac{i\Gamma}{2\pi}lnTe^{-i\gamma} \quad \cdots\cdots (3) .$$

Substituting $Re^{i\beta}$ for T in Eq. (3) after differentiation, the absolute velocity $|w(\beta)|$ on the circumference of the circle in the T-plane is given by

$$|w(\beta)|=2U_\infty\sin(\beta-\gamma)+\frac{1}{\pi R}\{\frac{Q_1(\sin\beta+\sin\varepsilon)+Q_2(\sin\beta-\sin\varepsilon)}{2(\cos\beta-\cos\varepsilon)}+\frac{\Gamma}{2}\} \quad \cdots\cdots (4) ,$$

where β is the angle in the T-plane (see Fig. 1).

Hence, Q_1 and Q_2 in Eq. (4) are determined from Eq. (4) by setting $|w(\beta)|=0$ at $\beta=\pm\delta$ on the circumference of the circle, because S_1 and S_2 are stagnation points. Substituting these values thus obtained for Q_1 and Q_2 in Eq. (4) again gives, some modifications, the absolute velocity in the T-plane;

$$|w(\beta)|=\frac{\cos\delta-\cos\beta}{\cos\varepsilon-\cos\beta}[2U_\infty\{\sin(\beta-\gamma)+\sin\gamma(\cos\varepsilon-\cos\delta)\}+\frac{\Gamma}{2\pi R}] \quad \cdots\cdots (5)$$

Therefore the absolute velocity $|w(\theta)|$ on the circular arc slit in the t-plane is obtained from Eqs. (1) and (5) as follows;

$$\frac{|w(\theta)|}{U_\infty}=\frac{1}{|f'(T)|}\frac{|w(\beta)|}{U_\infty}$$

$$=\frac{1-2\cos\beta\cos\delta+\cos^2\delta}{2U_\infty(\cos\varepsilon-\cos\beta)}[2U_\infty\{\sin(\beta-\gamma)+\sin\gamma(\cos\varepsilon-\cos\delta)\}+\frac{\Gamma}{2\pi R}] \quad \cdots\cdots (6)$$

Here, the radius R is taken to be $R=1/\sin|\delta|$ and the separation velocity K at the separation point is related to the base pressure coefficient C_{pb} obtained empirically by

$$K=(1-C_{pb})^{\frac{1}{2}} \quad \cdots\cdots (7)$$

Substituting these R and K in Eq. (6), the absolute velocity $|w(\theta)|$ in the t-plane is given by

$$\frac{|w(\theta)|}{U_\infty}=\frac{1-2\cos\beta\cos\delta+\cos^2\delta}{\cos\varepsilon-\cos\beta}[\sin(\beta-\gamma)+\sin\gamma\cos\varepsilon-\frac{\sin^3\delta\sin\gamma\cos\gamma}{K}] \quad \cdots\cdots (8)$$

The angular location of each source on the circumference is related to K, δ and γ by

$$\cos(\pm\varepsilon)=\frac{1}{K\mp\sin^2\delta\sin\gamma}[K\cos\delta+\sin^2\delta\{\sin(\delta\mp\gamma)\mp\frac{\sin^3\delta\sin\gamma\cos\gamma}{K}\}] \quad \cdots\cdots (9)$$

(double sign in the same order)

Then the pressure distribution along the circular arc slit $S_1'A'S_2'$ is expressed by

$$C_p=1-\{\frac{|w(\theta)|}{U_\infty}\}^2 \quad \cdots\cdots (10)$$

Here, the angles β in the T-plane and θ in the t-plane are related to each other as follows;

$$\sin(\pi-\theta)=\sin(\beta-\gamma)\cos\delta$$

$$+\frac{\sin^2\delta}{1-2\cos\delta\cos\beta+\cos^2\delta}\{\sin\gamma-2\cos\delta\cos\beta\sin\gamma+\cos\delta\sin(\beta+\gamma)\} \quad \cdots (11)$$

On the other hand, the angles δ and γ in the T-plane are related to the upper and lower separation points which are empirically determined on the circular cylinder, that is,

$$\delta=\frac{\theta_l-\theta_u}{4} \quad \text{and} \quad \gamma=\frac{\theta_l+\theta_u}{2}-180 \qquad (12)$$

Consequently the pressure distribution along the circular arc slit $S_1'A'S_2'$ in the t-plane can be calculated by using the present equations, Eqs. (7) to (12), if the three empirical values of θ_u, θ_l and C_{pb} are prescribed.

4 Experimental apparatus and method

The measurement of the circumferential static pressure distribution of the model cylinder was made with the same apparatus as reported before[3].

Fig. 2 shows a cross section of the model cylinder. The cylinder with an outer diameter of 100 *mm* is hollow and is inscribed by a small circular cylinder with a diameter of 25 *mm*. The height of the slot is constant, 0.55 *mm*, across the whole span and the outer surface of the cylinder is chromium-plated. The cylinder can be revolved about its axis to give a desired angular location of the slot θ_j. The cylinder has 55 static pressure holes at the mid-span, which are circumferentially distributed.

Fig. 3 shows the test section of the wind tunnel. The cylinder is mounted at the center of the test section and passes through two partition walls made of a transparent acrylic-resin plate of 10 *mm* thick and the tunnel side-walls.

The experiment was carried out by varying C_μ from 0 to 0.3 at the fixed values of the aspect ratio of the cylinder of eight and at the slot locations θ_j of 50° and 90°. In the case of $\theta_j=50°$, the experiment was made at the constant Reynolds number of 2.1×10^5 only and in the case of $\theta_j=90°$, the Reynolds numbers of the experiment were 1.4×10^5 and 2.1×10^5.

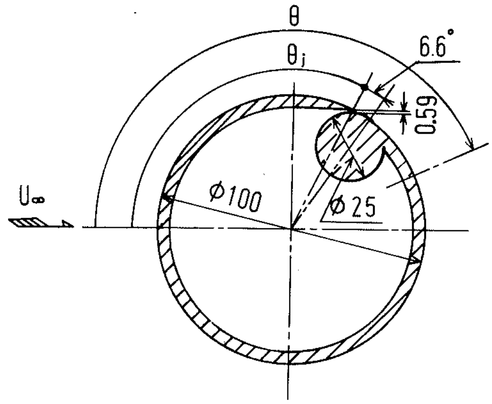

Fig.2 Cross section of the model cylinder.

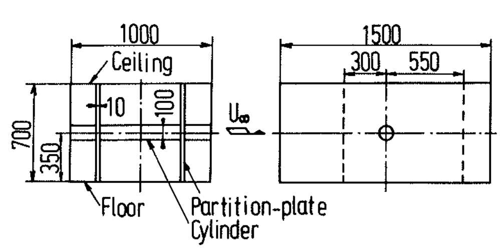

Fig.3 Test section of the wind tunnel.

5 Calculated results and discussion

Fig. 4 shows the typical examples of the calculated results of the circumferential static pressure distributions of the cylinder for a symmetrical ($C_\mu = 0$) and an asymmetrical ($C_\mu > 0$) flows with respect to the x-axis. This figure includes also the calculated distributions obtained from the Dunham's equation together with the experimental results.

When $C_\mu = 0$, the present method becomes the Parkinson-Jandali's one and the calculated distribution confirms this. Also the calculated distribution agrees fairly well with the experimental results.

When $C_\mu > 0$, the calculated distribution, in spite of the asymmetry of the flow, is fairly consistent with the experimental results over a wide range of θ. It is found from Fig. 4 that the present calculated result agrees better with the experimental results for the range of θ between the angular locations of the lowest pressure θ_m and the upper separation point θ_u in comparison with the result calculated by using the Dunham's equation. As a whole, the degree of consistency of the calculated result with the experimental results appears satisfactory except for the neighbourhood of the separation points. The discrepancy near the separation point is thought to be caused by the fall of the calculation accuracy there, because the effects of the boundary layer thickness and the vorticity in the wake on the flow past the cylinder are ignored in this theory.

On the other hand, the separation condition is described here, though about only one of the two separation points for convenience. If the condition of "smooth separation" by Woods[4] is applied to the flow at the separation point S_1', the separation velocity $K(\neq 0)$ at that point is determined uniquely as a function of only the angles of δ and γ in the T-plane. In this case, the curvature of the separation streamline is finite and the pressure coefficient C_p is continuous at the point S_1'. The static pressure distribution curve and the value of C_{pb}

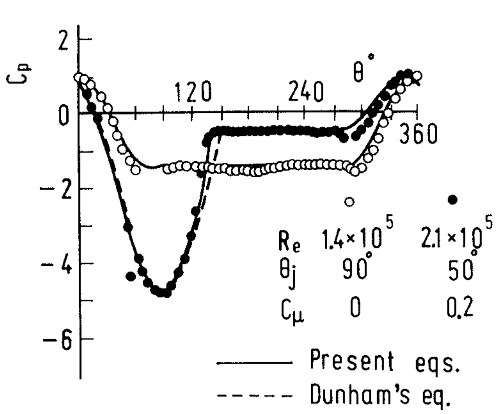

Fig.4 Static pressure distributions on the circular cylinder.

calculated in this case, however, were not so consistent with the experimental results. Ignoring the condition of "smooth separation", the pressure distribution was re-calculated for many given values of K, different a little from each other, in Eq. (7) and the best fitted curve with the measured values was chosen. Thus, for the optimum value of K, the calculated pressure distribution was best consistent with the experimental results. However, the pressure gradient $\partial C_p / \partial\theta$ became to be infinite, the curvature of the separation streamline was also infinite, and the base pressure coefficient C_{pb} was discontinuous at the separation point S_1'. Here, to avoid a singularity of the calculated pressure distribution curve at the separation point, the calculated values of θ_u and C_{pb} at the separation point were replaced with the experimental ones.

The same manipulation mentioned above has to be made at the other separation point S_2'.

6 Concluding remarks

An attempt was made to calculate the circumferential static pressure distribution of a circular cylinder with tangential blowing by the modified Parkinson-Jandali's method, extended to an asymmetrical flow. It was confirmed that the pressure distributions calculated by using the present method were better consistent with the experimental results over a wide range of θ in comparison with the distribution calculated by using the Dunham's equation. However the fall of the calculation accuracy in the neighbourhood of the separation point is unavaidable as far as the calculation of the pressure distribution is based on the free streamline theorem. The present method may easily be applied to other asymmetrical flows past bluff bodies.

References

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