Multi-objective Programming in Water Resources Development

by

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(Received May, 1980)

The paper deals with a typical type of water resources planning, i.e. water resources allocation on an areawide basis. This problem involves the reconcilation of conflicting interests among water users. The intended purpose of this study is to present two approaches to this multi-objective programming problem ; one is based on the Belenson method and another on the goal programming with L-type utility function.

With the Southern Part of Hyogo Prefecture as the study area, the advantages and disadvantages of the selected methods have been systematically compared. It has been shown that the two methods presented here may help the decision-maker systematically assess the promising alternatives, wherby the order of priority being explicitly articulated for the set of objectives and the resultant planning outputs being clearly illustrated for each objective.

1 Introduction

The intended purpose of this study is to present two possible multi-objective programming approaches to an inter-basin water resources allocation problem as defined later. Much work has already been done by the author but this study deals with a different type of inter-basin water resources allocation problem and presents a comparative analysis of the proposed two possible approaches. A case study will be conducted for the Southern Part of Hyogo Prefecture which comprises five major river basins running in parallel and which is one of the most industrialized and urbanized areas in Japan.

Close examinations of the computational results obtained from the model application to the region will follow. The paper closes with some assessment of the applicability of the proposed methodologies and needed interface devices to be developed to supplement the model.

2. Problem Definition

Bearing in mind a typical water management in the metropolitan areas of Japan,

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we will work with the following predetermined basic framework.

(1) Our major concern is with the development of a water utilization system on an areawide basis.

(2) The study region consists of several river basins.

(3) The basins are classified into two categories. One is those basins where the water demand on the stream exceeds the available supply of fresh water ; the other those where, to the contrary, the available supply of fresh water exceeds the water demand.

(4) Two modes of water sources are available, fresh water to be developed by throwing dams across the streams, and recycling renovated wastewater by adding an extra treatment process (called tertiary treatment) to the ordinary process.

(5) The wastewater which has undergone tertiary treatment is partially or totally supplied for exclusive use in industry, thereby the renovated water being assumed to have been blended beforehand with the industrial water purified at the purification plant and conveyed to industry through a common pipeline.

We must observe here that the blending of industrial water with renovated water results in a degraded quality of water which would not fit certain types of industrial processes. Accordingly, we need to identify the amount of those demands for particular industrial uses which require a higher level of quality than that of the blended water. This mechanism will be incorporated into the model. We shall analyze this mechanism in the subsequent section.

(5) If necessary, channels will be constructed to convey fresh water from one stream to another.

(6) We are involved in the conflicts of the following two different objectives. The regional water agency in charge of regional water management seeks for an alternative that guarantees the most economical system on the entire region basis. But the agency is also asked to conserve as much as possible the local river systems, namely, the closed-basin water utilization in the individual river basin. The former objective represents the maximum attainment of economic efficiency and the latter the maximum attainment of river environment conservation. These two objectives would conflict each other if we pursue the full attainment in either of the two objectives. In this respect the agency has to develop some methodology for finding a best compromise. This problem is called a multi-objective programming problem.

(7) Let us call the former objective "efficiency objective" and the latter "conservation objective". More specifically the efficiency objective is formulated as minimizing the total associated costs and the conservation objective as minimizing the total amounts of water to be diverted from one basin to another.

(8) The facilities to be explicitly considered in each basin are a set of dams to be constructed on the farthest upstream, inter-basin channels for streamflow diversions to be built between two adjacent basins, two filtration plants, one for industrial use

and another for domestic use, a wastewater treatment plant and a tertiary treatment plant.

(9) In each stream water quality is to be regulated to meet the prescribed standard (in terms of BOD ppm.) at the check point located farthest downstream.

(10) The development ought to be made to meet the demands of industrial and domestic use projected for a given time in future. We assume that the total supply be equated the total demand. When supply equates demand, the terms "supply" and "demand" will be used synonymously in this paper.

(11) Minor assumptions will be referred to later when specifications become necessary.

3. Maximum Demand for Blended Water

On the basis of the data on the structure of the industrial water use in the Southern Part of Hyogo Prefecture and taking into account the industrial development plan for 1985, Yoshinaga, Fujimoto, Okada and Yoshikawa have estimated the maximum amount of the projected water demand which can be covered in light of quality by blending industrial water purified at a filtration plant with the renovated wastewater at a given blending ratio[1)2)3)] (see **Fig. 1**). Generally, this maximum amount, f, which we call the "maximum demand for blended water", is given as a function of blending ratio, r.

$$f = f(r) \qquad \cdots\cdots (1.1)$$

where blending ratio r is defined in terms of industrial water supply, S^I and the renovated water supply for industrial use, U.

$$r = U/(S^I + U) \qquad \cdots\cdots (1.2)$$

It has been found that f can be approximated by the following equation[3)] :

$$f = b/(1 + ar) \qquad \cdots\cdots (1.3)$$

where a and b are parameters whose values differ for river basins.

Equations (1.2) and (1.3) will be incorporated into the model to be set up in the subsequent section.

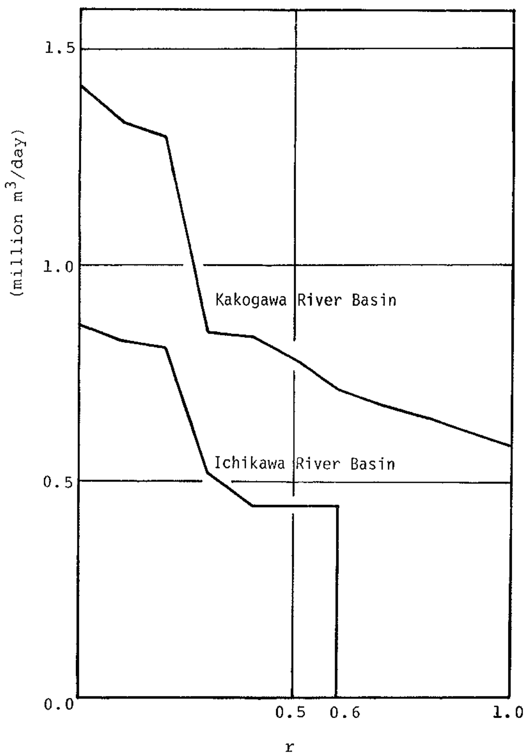


Fig. 1 Maximum demand for blended water vs. blending ratio

4. Model Formulation

1) Symbols

The following variables will appear in the model formulation.

X_{ij} : amount of fresh water to be stored in the jth dam ($j = 1, \ldots, m_i$ in the order of location downwards from the farthest upstream) on river i ($i=1, \ldots, n$).

R_i : that portion of $\sum_{j=1}^{mi} X_{ij}$ which is reserved for improving streamflow quality in river i.

$Y_{ki}(Y_{ik})$: amount of streamflow to be diverted from river $k(i)$ to $i(k)$.

$Y_{ki}^S(Y_{ik}^S)$; that portion of $Y_{ki}(Y_{ik})$ which is withdrawn from river $i(k)$.

$Y_{ki}^Q(Y_{ik}^Q)$: the remaining portion of $Y_{ki}(Y_{ik})$ which contributes to an improvement in the streamflow quality in river i.

S_i^M : total municipal water supply (amount of municipal water to be purified at the municipal water filtration plant in river basin i).

S_i^I : total industrial water supply (amount of industrial water to be purified at the industrial water filtration plant in river basin i).

W_i^I : effluent to be discharged into the water body i from the wastewater secondary treatment plant.

S_i^R : total amount of renovated wastewater at the tertiary treatment plant.

T_i : effluent to be discharged from the tertiary treatmant plant into the water body i.

U_i : amount of renovated wastewater to be reused for industrial use in river basin i.

I_i^M : municipal water supply for industrial use (complementary supply of municipal water for industrial use).

R_i : amount of streamflow to be reserved for maintaining the quality standard.

The following parameters are used :

c_{ij} : maximum capacity of dam j on river i.

s_i^P : total amount of wastewater currently being treated in river basin i.

d_i^M : total water demand for municipal use in river basin i.

d_i^I : total water demand for industrial use in river basin i.

a_i , b_i : parameters in the maximum demand function, f_i for blended water in river basin i.

2) Technical and physical constraints

In addition to the nonnegativity conditions to hold for all the variables, the following constraints must be satisfied.

The maximum capacity condition for each dam reads :

$$X_{ij} \leqq c_{ij} \quad (i = 1, \ldots, n \; ; \; j = 1, \ldots, m_i) \qquad \cdots\cdots(2.1)$$

The following condition holds for the amounts of streamflow to be diverted and for the amounts of water to be impounded and withdrawn in each river basin :

$$\sum_{k\epsilon K_i} Y_{ki} - \sum_{k\epsilon K_i} (Y^s_{ki} + Y^s_{Ki}) = 0 \, (i = 1, \cdots, n) \quad (2.2)$$

$$\sum_{j=1}^{mi} X_{ij} + \sum_{k\epsilon K_i} (Y_{ki} - Y_{ik}) - (S^M_i + S^I_i) - R^I = 0 \, (i = 1, \cdots, n) \quad \cdots\cdots\cdots\cdots\cdots\cdots \, (2.3)$$

where K_i stands for the set of those rivers adjacent to river i.

The water demand and supply conditions are expressed as :

$$S^M_i - (d^M_i + I^M_i) = 0 \quad (i = 1, \cdots, n) \quad (2.4)$$

$$S^I_i + U_i - (d^I_i - I^M_i) \quad (i = 1, \cdots, n) \quad (2.5)$$

$$S^R_i + W^I_i - (s^P_i + S^M_i + S^I_i + U_i) = 0 \quad (i = 1, \cdots, n) \quad \cdots\cdots\cdots\cdots\cdots\cdots \, (2.6)$$

$$U_i + T_i - S^R_i = 0 \quad (i = 1, \cdots, n) \quad \cdots \, (2.7)$$

where I^M_i, complementary supply of municipal water for industral use represents the amounts of water to be supplied from municipal water if the quality of the blended water is not adequate enough for the water uses. It is assumed here :

$$S^I_i + U_i \leqq f_i \quad (i = 1, \ldots n) \cdots\cdots (2.8)$$

which means that the total amounts of blended water supply cannot exceed the maximum demand for blended water in each river basin (see **Fig. 2**).

Substituting Equations (1.2) and (1.3) into (2.8), the above condition is rewritten as :

$$S^I_i + U_i + a_i U_i - b_i \leqq 0 \quad (i = 1, \ldots, n) \quad \cdots\cdots\cdots\cdots (2.9)$$

Finally, the quality of each stream is required to satisfy the the BOD standard, τ_i prescribed for each river basin.

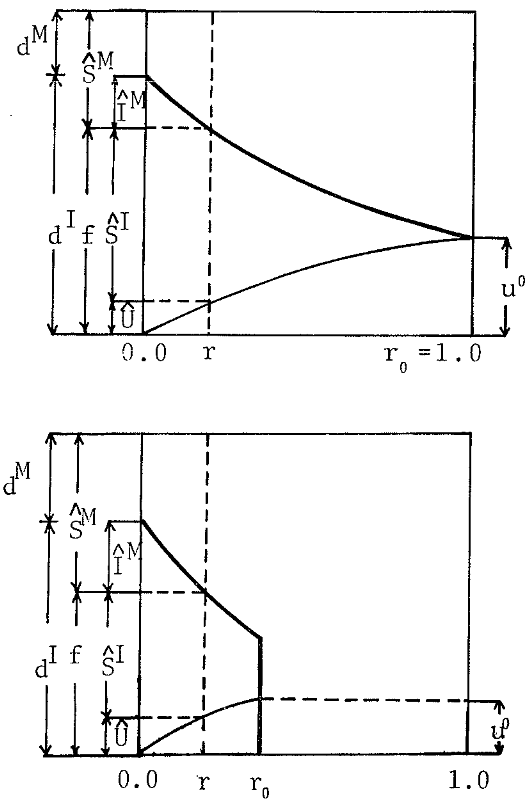


Fig. 2 Maximum demand for blended water vs. blending ratio (approximation)

$$\tau_i \geqq (\gamma_i q_i + \alpha W_i + \beta T_i + \sum_{k\in Ki} \gamma_k Y^Q_{ki} + \delta_i R_i) / (q_i + W_i + T_i + \sum_{k\in Ki} Y^Q_{ki} + R_i),$$
$$(i=1, \cdots, n) \quad \cdots\cdots\cdots\cdots \quad (2.10)$$

where q_i represents the minimum streamflow requirement to be reserved for the current normal streamflow conditions, α and β, the average quality of treated wastewater at the wastewater treatment plant (these values being assumed to be constant for each river basin), γ_i , the quality of q_i , and δ_i , the average quality of the fresh water to be developed in river basin i.

3) Objectives

According to our problem definition, we shall formulate both the efficiency and the conservation objective in the following manner :

The efficiency objective reads :

$$\textbf{Minimize} \sum_{i=1}^{n} \sum_{j=1}^{m1} e^1_{ij} (X_{ij}) + \sum_{i=1}^{n} \sum_{k\in Ki} e^2_{ik} (Y_{ik}) + \sum_{i=1}^{n} \Big\{ e^3 (S^M_i) + e^4 (S^I_i)$$
$$+ e^5 (S^I_i + S^M_i) + e^6 (S^R_i) \Big\}, \quad \cdots \quad (2.11)$$

where $e^1_{ij}(X_{ij})$ represents the cost function of dam i on river j, $e^2_{ik}(Y_{ik})$ the cost function of inter-basin canal (ik), e^3 (S^n_i) and e^4 (S^I_i) the cost functions of purification plants, the former being municipal and the latter industrial, e^5 $(S^M_i + S^I_i)$ the cost function of the joint wastewater secondary treatment plant, and e^6 (S^R_i) the cost function of the tertiary treatment plant.

On the other hand the conservation objective is expressed as :

$$\textbf{Minimize} \sum_{i=1}^{n} \sum_{k\in Ki} Y_{ik} . \quad \cdots\cdots\cdots\cdots \quad (2.12)$$

Now we have formulated our multi-objective programming problem. In the section that follows let us consider two promising methodologies in solving the above formulated model.

5. The Belensom Method and The Goal Programming with L-Type Utility Function

5.1 Multi-objective Programming Problems

A general description of the multi-objective programming problem is given as follows :

For a set of given objectives* :

$$\textbf{Maximize} \quad f_1(\mathbf{X}) = \mathbf{C}^1\mathbf{X},$$
$$\cdots\cdots\cdots\cdots,$$
$$\textbf{Maximize} \quad f_k(\mathbf{X}) = \mathbf{C}^2\mathbf{X}, \quad (k=1, \cdots, m) \quad \cdots\cdots\cdots\cdots \quad (3.1)$$
$$\cdots\cdots\cdots\cdots,$$
$$\textbf{Maximize} \quad f_m(\mathbf{X}) = \mathbf{C}^3\mathbf{X},$$

* A similar discussion applies to the problem of minimizing the objectives.

subject to the technical and physical constraints as :

$$\begin{aligned} &\mathbf{AX} \geq \mathbf{b} \\ &\mathbf{X} \geq \mathbf{0} \\ &\mathbf{X} = (x_1,\ x_2,\ \cdots,\ x_q) \\ &\mathbf{C}^k = (c_1^k,\ c_2^k, \cdots,\ c_q^k) \\ &\mathbf{b} = (b_1,\ b_2, \cdots\cdots,\ b_p) \\ &\mathbf{A} = \begin{bmatrix} a_{11} \cdots\cdots a_{1q} \\ \cdots\cdots\cdots\cdots \\ a_{p1} \cdots\cdots a_{pq} \end{bmatrix} \end{aligned} \qquad \cdots\cdots\cdots\cdots (3.2)$$

Let us assume here that all the objectives are conflicting or "non-trivial", which means that in maximizing all the objectives one objective cannot be fully attained without regulating the attainment of the rest of the objectives in one way or another. That is, there does not exist any optimal solution $\mathbf{X}_{opt}$ which is common to each objective function. In this case it is necessary to define a different concept for a certain acceptable solution set. This kind of solution is called "efficient solution" in mathematical terms and is defined as follows :

Letting S represent the feasible solution set, a point $\mathbf{X}^* \in S$ is known as an efficient solution if there does not exist another feasible solution $\mathbf{X} \in S$ such that :

$$f_k(\mathbf{X}) \geq f_k(\mathbf{X}^*) \text{ for all } k=1, \ldots, m. \qquad \cdots\cdots\cdots\cdots\cdots (3.3)$$

and

$$f_k(\mathbf{X}) \neq f_k(\mathbf{X}^*) \text{ for at least one k}. \qquad \cdots\cdots\cdots\cdots\cdots (3.4)$$

One typical example of efficient solutions is such an optimal solution which solves a single-objective optimization with a particular objective function and the rest of the objective functions left out from the original problem. This solution is optimal for the particular objective function but not necessarily optimal for the rest of the objective functions. This solution can be considered as an efficient solution for the given multi-objective programming problem.

The multi-objective programming problem in general terms is characterized by the two mechanisms : one which produces a set of efficient solutions ; and one which locates a single efficient solution among them as the best compromize solution (alternative). Many techniques which have been developed for solving different types of multi-objective programming problems can be broken down into two categories, depending on the way the latter mechanism is dealt with.

A class of techniques which fall into the first category deals with only the former mechanisms which produces a set of efficient solutions, without reference to the latter mechanism which locates a specific efficient solution as the best compromise. These types of techniques commit the latter function totally to the outside of the model, namely some other submodels or the decision-maker. The Belenson Method[4] and the SWT Method due to Haimes[5] and many others categorized as interactive man-machine techniques[6)7)] are considered to be among these types of techniques.

The other class of techniques which fall into the second category has the mechanism of explicitly quantifying the trade-offs among objectives automatically incorporated. Goal programming is a typical example of the latter type[8]. There are many variants of goal programmings which have already been developed. Amongst them one of the most promising technique is that of Fushimi and Yamaguchi[9] who have shown that if an L-type utility function which represents a basic form of unspecifiable utility functions for multiple objectives at hand is incorporated into a conventional type of goal programming formulation, the resultant model produces such a solution that is characterized by a good balance in the attainment of each objective.

In this study we shall approach our multi-objective problem in two ways—by use of the Belenson Method (as a representative technique of the first category) and the goal programming with L-type utility function incorporated (as a representative technique of the second category). Then we will make a comparative analysis of the applicability of the two approaches with the Southern Part of Hyogo Prefecture as the study area.

5-2 The Belenson Method

The Belenson Method proceeds with the construction of a "payoff table" which is developed by solving

$$\textbf{Maximize}\ f_k(\mathbf{x}) = \mathbf{C}^k \mathbf{X} \text{ for } k = 1, \ldots, m \quad \cdots\cdots\cdots\cdots\cdots(3.5)$$

subject to

$$\mathrm{X} \ni \mathrm{S}.$$

By solving this problem also for $k=2,\ldots,m$, we get m optimal (efficient) solutions $\mathbf{X}^*(k)$ $(k=1,\ldots,m)$. This $\mathbf{X}^{*(k)}$ $(k=1,\ldots,m)$ gives by definition the maximum value of the kth objective, f_{kk}, namely, $f_{kk}=f_k\,(X^{*(k)})$. By analogy, f_{jk} $(j=1,\ldots,m\ ;\ k=1,\ldots,m,\ j\neq k)$ are defined for the $(m-1)$ objective functions f_j other than f_k. That is, $f_{jk}=f_j\,(\mathbf{X}^{*(k)})$. By calculating f_{jk} for $j=1,\ldots,m$ and $k=1,\ldots,m$, we can construct a payoff table shown in **Table 1**. The values of the m objective functions for the efficient solution $\mathrm{X}^{*(k)}$ appear in the kth column in this table.

Table 1 Payoff matrix

j \ k	1	2	3
1	f_{11}	f_{12}	f_{13}
2	f_{21}	f_{22}	f_{23}
3	f_{31}	f_{32}	f_{33}

It is generally the case that disparities exist between the magnitude of the values generated by the various objective functions and that the unit of measurements are not common for each of the objective functions. In order to compensate for these discrepancies let us normalize the payoff entries as follows.

On dividing f_{jk} by f_{jj} which is the maximum value that f_j can achieve for $j=1, ..., m$, new payoff table entries can now be formed :

$$f'_{jk}=f_{jk}/f_{jj}=\mathbf{C}^j\mathbf{X}^{*(k)}/f_{jj} \quad \cdots\cdots\cdots \quad (3.6)$$

for $j=1,...,m$ and $k=1,...,m$.

Another problem may be encountered if $f_{jk} \leqq 0$ for all k and for at least one j. This situation appears to be especially detrimental if $f_{jj}=0$ for some j since normalization cannot be performed. The procedure for dealing with this problem is to add a sufficiently large fixed constant, K to all the entries in the payoff table which cannot alter the outcome of the computation. In order to provide for consistency when applying the algorithm, the determination of the value for the constant K will be performed as follows :

If for at least one j, $f_{jk} \leqq 0$ for all $k=1,...,m$, then :

$$K = -\min_{k,j} f_{jk}. \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (3.7)$$

Otherwise

$$K = 0. \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (3.8)$$

Therefore the resultant payoff table entries are given as :

$$f'_{jk} = (f_{jk}+K)/(f_{jj}+K) \quad (j, k=1,...,m) \quad \cdots\cdots\cdots\cdots (3.9)$$

With the entries in the payoff table obtained as such, we proceed to integrate the multiple objectives into a single one in the following way. The idea is that we assign weights λ_k to f'_{kj} $(k=1,...,m)$ to obtain a synthetic value, E_j for each objective function where

$$E_j = \sum_{k=1}^{m} \lambda_k f'_{kj} \quad (j=1, \cdots, m), \quad \cdots \quad (3.10)$$

$$\sum_{k=1}^{m} \lambda_k = 1. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (3.11)$$

Notably the resulting value E_j for each of the objective functions can be viewed as an expected value the efficient solution $\mathbf{X}^*$ takes on.

In light of these considerations the problem of determining the best weighting values can be interpreted as a mixed strategy game. The theory of mixed strategy game shows that a stable compromise can be made if and only if it holds

$$E_1 = \cdots\cdots E_j = \cdots\cdots = E_m . \quad \cdots\cdots\cdots\cdots\cdots\cdots (3.12)$$

That is, the weighting values are obtained by solving a set of linear equations that follow.

$$\sum_{k=1}^{m} \lambda_k f_{k1} = \cdots\cdots = \sum_{k=1}^{m} \lambda_k f_{kj} = \cdots\cdots = \sum_{k=1}^{m} \lambda_k f_{m} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (3.13)$$

$$\sum_{k=1}^{m} \lambda_k = 1. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (3.14)$$

For a set of weighting values so obtained, let us define a representative objective function F as follows :

$$F^{(1)}(\mathbf{x}) = \sum_{k=1}^{m} \lambda_k f_k(\mathbf{x}). \quad \cdots\cdots\cdots \quad (3.15)$$

Geffrion showed that the solution of a new optimization problem which is defined by this representative function and the set of technical and physical constraints, is also an efficient solution to the original multi-objective programming problem.[8) This new efficient solution is denoted by $\mathbf{X}^{*(m+1)}$.

Here let us commit the evaluation of this new efficient solution to the decision maker by asking him to make judgment as to whether or not this alternative can be regarded as most acceptable. If he regards it as most acceptable, the computation terminates and we employ the solution, $\mathbf{X}^{*(m+1)}$ as the compromise solution. Otherwise, we ask the decision maker to identify one objective function on which he places the least priority, and we replace it by the representative objective function, $F^{(1)}(\mathbf{X})$ to obtain a renewed payoff table.

The iteration goes on as before until the newest efficient solution, $\mathbf{X}^{*(m+v)}$ is identified as most acceptable by the decision maker, where v represents the number of iteration and $1 \leqq v \leqq m$. That is, the maximal number of iteration is identical to that of objective functions, because such a replacement by the newest representative objective function can be produced at largest m times as many as the number of the objective functions.

This method developed by Belenson is characterized by the repetitive interactions between the computation on the analyst's part and the evaluation on the decision maker's part. In other words the algorithm per se cannot automatically locate one solution as the best compromise (the most acceptable) alternative but it can do so only with the aid of the decision maker.

5-3 Goal programming with L-type utility function

One essential difference between the Belenson Method and goal programming is that the latter treats all the objectives as if they were constraints to be added to technical and physical constraints. That is, the original multi-objective programming problem is formally converted to a single-objective programming problem in the following way.

The procedure begins with specifying two levels for each of the objectives, that is, the permissible and the satisfactory level, the former being a critical limit to one particular objective such that any level below that would not be accepted by the decision maker, and the latter a tentative upper limit such that any level equal to or beyond that would be regarded as satisfactorily acceptable to the decision maker.

Our next task is how to articulate a utility function for a set of given attainment levels of the objectives. To take an example of a two-objective problem, let us assume a space spanned by the two orthogonal axes representing the attainment levels of the two corresponding objectives. (This space is called goal space).

Economists have theoretically shown that a utility function takes a form of downward convex against the two orthogonal axes representing the two objectives. But when it comes to the articulation of the utility function for a set of specific objectives in practical considerations, this theory gives us no more than that. This means that we have to practically manage to locate this function without any reference to this theory. Since this is not an easy task for the time being because of limited data collectability, a second best approach is to approximate the form of any utility function by a set of L-formed contours as shown in **Fig. 3**. This form of utility functions will be called L-type utility functions. The following discussion will lend support to the validity of the L-type utility function.

Let us assume here a directed line emanating from the most downward point in the goal space corresponding to the permissible levels of the concerned objectives to the most upward point corresponding to the satisfactory levels. (This line is called goal vector.) In many practical planning problems the planner is asked to produce such an alternative that would guarantee well-balanced attainments of the objectives, rather than to provide for any alternative that would lead to high attainment levels for some of the objectives and relatively low ones for the rest of the objectives. In this respect the approximation to the utility function by an L-type function would be promising.

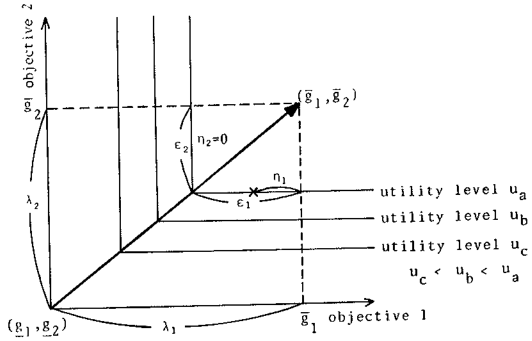


Fig. 3 L-type utility function

The goal vector can be considered as the direction in which the attainment of each of the objectives should be improved. In other words any point on this vector represents an alternative with well balanced attainment in every objective, not necessarily most acceptable though. In this sense it seems reasonable to assume that the contours of the L-type utility function are bound to have its corner of inflection on the goal vector.

Based on the above discussions a general formulation of the goal programming with L-type utility function is given as follows :

Letting ε_i and η_i represent two kinds of deviational variables standing for the degrees in which each of the attained objectives deviate from its satisfactory level, and $\bar{g}_i$ and $\underline{g}_i$ the satisfactory and the permissible level for objective i ($i = 1,..., m$), the goal programming problem in a general form is formulated as follows :

<objective function>

Minimize ε_{i_0} (i_0 being any one of $i = 1, ..., m$) ....(3.16)

<goal constraints>

$$c^i\mathbf{X} + \varepsilon_i - \eta_i = \bar{g}_i \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (3.17)$$

$(\varepsilon_i, \eta_i \geqq 0,\ i = 1, \cdots, m$;

$\mathbf{X} = (x_1, x_2, \cdots, x_q)'$

and $\mathbf{C}^i = (c_1, c_2, \cdots, c_q)$.

$$\mathbf{C}^i\mathbf{X} \geqq \underline{g}_i \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (3.18)$$

$$\frac{\varepsilon_1}{\lambda_1} = \frac{\varepsilon_i}{\lambda_i}\ (i = 2, \cdots, m) \quad \cdots\cdots\cdots\cdots\cdots \quad (3.19)$$

Equation (3.19) represents the condition that the L-type utility function should have its corner of inflection on the goal vector and λ_i ($i = 1, ..., m$) are equal to $\bar{g}_i - \underline{g}_i$.

<technical and physical constraints>

$\mathbf{AX} \geqq \mathbf{b}$,

$(\mathbf{X} \geqq \mathbf{o}$, and $\mathbf{b} = (b_1, b_2, \cdots, b_p))$ (3.20)

It can be easily shown that the solution to the above problem is among a set of efficient solutions. It must also be observed that this algorithm leads automatically to a single solution without any intervening articulation by the decision maker. This does not mean that the algorithm is totally independent of the decision maker. Instead, his role is to articulate the satisfactory and the permissible levels for each of the objectives with an aid of additional information which would be available from another submodel and the expertise.

The decision maker is also asked to judge whether the solution is an acceptable one from the points of view not explicitly taken in the model.

6. Case Study on the Southern Part of Hyogo Prefecture

The southern part of Hyogo Prefecture is selected as the study area to which the above formulated model will be applied.

The conceptualized system of interbasin water resources development is diagrammed in **Fig. 4**, which shows that the interbasin system consists of five river basins, namely, the Chigusa, Ibogawa, Yumesaki, Ichikawa and Kakogawa River Basins.

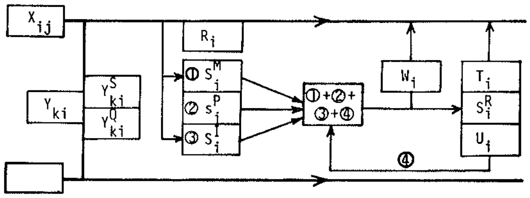


Fig. 4 Diagrammatic representation of the water utilization system (on a single basin)

6.1 Input Data

We set the target year as 1985 and the water demands for this year were projected by a regression model developed by the author et al. The projected water demands are listed in **Table. II.**

Table II Model inputs (1)

(m³/day)

River Basin	total water demand	
	municipal	industrial
Kakogawa	1,427,400	580,000
Ichikawa	953,200	265,500
Yumesaki	179,400	49,000
Ibo	118,200	19,900
Chigusa	445,100	32,500

The unit construction costs for both dams and canals are all based on the data provided and authorized by a certain consulting company. Some of the cost data are listed in **Table. III** and **IV**. Prior to the computations on the model, we preplanned the following cases, taking into account the range of probable variation in the water demand forecast and allowing for the variety of water quality regulation levels for each quality check points.

Let **Case A—1** stand for the standard case where calculation is made for those parametric values as shown in **Tables II** and **V**, **Cases A—2** and **A—3** those cases where the water demand is assumed to be less than the forecast by 5 and 10 percent, respectively. **Cases B** and **C** represent those cases where the minimum streamflow requirement is assumed to be less than that for the standard case by 5 and 10 percent, respectively. **Cases B—1, B—2** and **B—3** correspond to **Cases A—1, A—2** and **A—3.** So do **Cases C—1, C—2** and **C—3.**

It must be noted here that our approach will be slightly different from the approach as Belenson proposed. We merely produce several different efficient solutions for each of the above cases, and provide for a set of efficient solutions for the dicision maker, leaving it open as to which one is to be selected as the most accept-

Table III Model inputs (2)

dam No. / River Basin	Cost of developing reservoir 1	2	3	4	5	6
Kakogawa R.	8.67	9.65	10.18	13.03	15.63	17.03
Ichikawa R.	7.00	15.15	15.27	41.10		
Yumesaki R.	3.72	27.07				
Ibo R.	5.82	12.31	17.81	38.86		
Chigusa R.	6.51	7.83	22.00	26.63	27.69	37.33

(yen/m³/day)

Table IV Model inputs (3)

Scale (1000m³/day)	Cost of tertiary treatment (yen/m³/day)
0 ～ 50	25.04
50～150	22.64
150～500	20.14
500～	14.00

Table V Model inputs (4)

River Basin	streamflow to be resevred	streamflow quality standard
Kakogawa R.	751.7(10^3 m³)	3.5(BODppm)
chikawa R.	216.0	2.4
Yumesaki R.	70.7	1.8
Ibogawa R.	380.2	1.9
Chigusa R.	257.5	1.0

table one among them. This idea bases its ground on the fact that in many practical situations it would be more productive and educative for the decision maker to make a choice among a broad set of those alternatives including those which would not have been obtained if the decision maker would have intervened in the preceding process in order to single out one alternative.

In this respect, let alternative **A–1** (**E**) represent such an efficient solution for **Case A –1** that is obtained by solving a single-objective optimization problem in which the efficiency objective is set as the explicit objective function while the conservation objective is excluded from the model. Likewise, let alternative **A–1** (**C**) represent an efficient solution that is obtained by solving another single-objective optimization problem in which the conservation objective is set as the explicit objective function with the cost objective excluded from the model.

Furthermore let alternative **A–1** (G_1) represent such an alternative that is obtained by applying the Belenson method to **Case A–1**, that is, by solving another optimization problem which is derived from the pay-off table obtained on the basis of alternatives **A–1** (**E**) and **A–1** (**C**). Provided that this alternative is not regarded as most acceptable by the decision maker and that the lowest priority is given to either of the efficiency or conservation objectives and that the Belenson Method is applied to either of the two cases, let us denote the corresponding solutions by alternative **A–1** (G_2) or **A–1** (G_3), respectively.

In addition, let the solution to the goal programming model for **Case A–1** be denoted by alternative **A–1** (**GP**). This symbolic system also applies to **Cases B** and **C.**

6.2 Computation

The computation results for all the cases are illustrated in **Figs. 5** to **9. Fig. 5** illustrates a diagrammatic representation of the computation result for **Case A – 1** (the standard case). To begin with, let us focus on the result for **Case A–1** for the moment and study its characteristic features.

6.3 Standard Case

Let us first analyze alternative **A – 1** (**E**) which can be considered the alternative which mainly accounts for economic efficiency.

The following may be readily understood from the result :

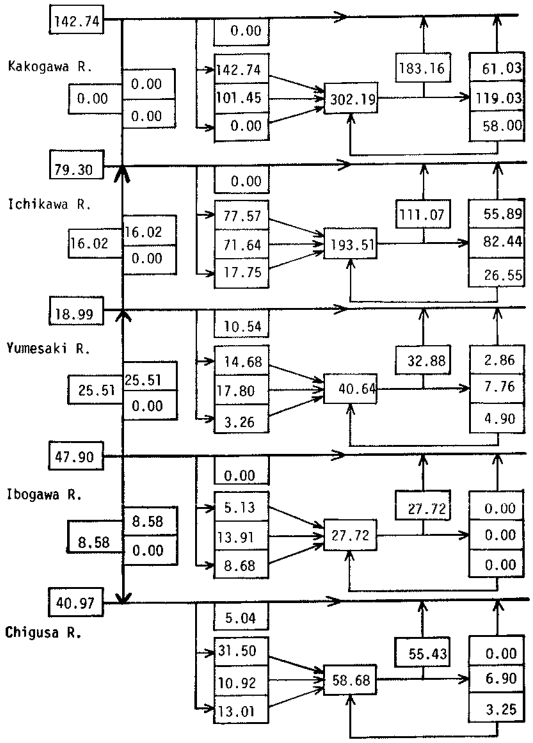


Fig. 5 Computation results (Case A-1 (E))
(unit : 10^4 m^3 /day)

(1) The Ibogawa River is the only stream from which streamflow is diverted to the other rivers. The total amount of diverted streamflow is found to be 341,000㎥/day, of which 160,000㎥/day goes to the Ichikawa River, 95,000㎥/day to the Yumesaki River, and 86,000㎥/day to the Chigusa River (see **Fig. 5**).

(2) The quality of streamflow is assumed to be equal to the prescribed standard of 10 BOD ppm for all the rivers except for the Ibogawa River where it was found to be 7.42 BOD ppm.

(3) The mode of controlling streamflow quality is found to differ among the rivers. On the Yumesaki River both some portion of the fresh water developed by the dams to be constructed on its basin and some portion of renovated wastewater treated at the tertiary treatment plant contribute to the improvement in the streamflow quality. In the Kakogawa and Ichikawa Rivers the streamflow quality is mitigated exclusively by wastewater reclamation. On the contrary, the Chigusa River achieves its streamflow regulation by discharging a portion of fresh water

into the stream. Notably, quite different from the other rivers, the Ibogawa River can attain 7.42 BOD ppm by discharging into the stream all of the wastewater disposed in the secondary treatment process.

(4) The wastewater reclamation is implemented on each of the rivers. One exception to this is the Ibogawa River which is found to dispense with any reclamation of wastewater.

(5) Let us define the degree of fresh water coverage as the ratio of the actual amount of fresh water to be developed by dams on each of the rivers to the water demand there. The ratios are found to differ much for the rivers. That is, the degree is found to be very high for the Ibogawa River and much lower for the rest of the rivers.

(6) These findings are considered to have been derived from the following a priori conditions.

  (i) Given that the degree of potential freshwater availability is defined for each river as the ratio of the total amount of potentially available fresh water on the river to the water demand there, this degree is found to be exceptionally high, namely 5.45 for the Ibogawa River, while it is found to range from 0.57 to 1.53 for the rest of the rivers. It is also true that dam construction is less expensive on the Ibogawa River than any other rivers.

  (ii) The minimum streamflow discharge of the Ibogawa River is relatively large as compared with the water demand there, whereas it is relatively small on the rest of the rivers. Furthermore the quality of the fresh water to be developed by the dams located farthest upstream is assumed to be relatively good, namely 1.9 ppm, which forms a striking contrast with the other rivers where it is assumed to be relatively bad.

(7) To restate, this type of economic-efficiency oriented approach was found to result in an intensive construction of dams on those rivers where the construction costs are much less and there is a larger amount of streamflow available than in the rest of the river basins ; thus leading to an imbalance in the degree of fershwater coverage for each of the rivers.

Then proceed to the analysis of alternative **A−1** (**C**) which is characterized by the we conservative utilization of the conventional closed basin systems. In comparison with alternative A−1 (E), the following features seem to deserve attention :

(1) The Ichikawa River is the only river which diverts some streamflow from the two adjoining rivers, namely, the Kakogawa and Yumesaki Rivers. The amount of streamflow accounts for 99, 700 m^3/day which is equal to the quantity of water which could not be developed otherwise even by renovating all portion of wastewater because there is a restriction on the quality of water supply.

(2) The quality of streamflow in each of the rivers was found to be as high as that for alternative **A−1** (**E**). The mode of controlling streamflow quality on each river basin is different only in that the streamflow quality is attained exclusively by the tertiary treatment system in all the river basins except the Ibogawa River where no tertiary treatment was found to be necessary.

(3) The total amount of renovated wastewater is equal to that for alternative **A − 1** (**E**). These features are also considered to have been derived from the predetermined conditions as pointed out before.

(4) To restate, the minimization of the amount of water to be diverted from one stream to another would result in the implementation of independent utilizations of the individual river system plus complementary inter-basin streamflow diversions to meet the absolute amount of shortage in supply which could not be covered otherwise. The resultant system is also characterized by a well-attained balance of the degree of fresh water coverage for each of the rivers.

With these findings obtained above, let us now study alternative **A−1** ($\mathbf{G}_1$) which is considered as the initial product derived from the application of the Belenson Method to the reconcilation of the preceding two alternatives (see **Fig. 6**).

The following may be readily understood :

(1) The water utilization system is basically the same in form as that of alternative **A − 1** (**C**) ; which means that the Ichikawa River is the only stream which receives some water diverted from the adjoining streams, the Kakogawa and Yumesaki Rivers. The mere defference is in the amount of water to be diverted, which accounts for 160,000 m³/day in this alternative—as much as that for alternative **A−1** (**E**) and more than that for alternative **A − 1** (**C**) by 60,000 m³/day.

(2) Let us define the attainment level for objective j as the ratio $(f_{jk_j} - f_j(\mathbf{X}^{*(m+v)})/(f_{jk_j} - f_{jj})$ where it holds

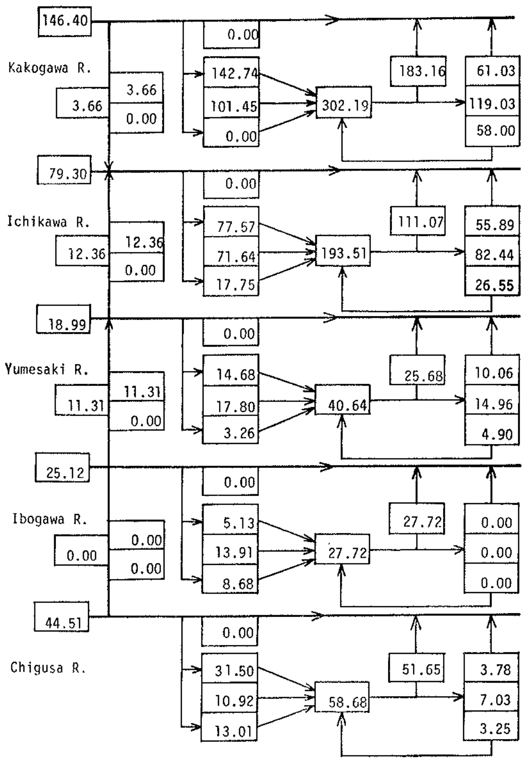


Fig. 6 Computation results (Case A-1 ($\mathbf{G}_1$))
(unit : 10^4 m³ /day)

$f_{jj}=f_j\ (\mathbf{X}^{*(m+v)})$ and $f_{jk_j} = \min\limits_k f_{jk}$.

This ratio indicates the degree in which a particular objective j has approached its highest level f_{jj} (or satisfactory level in terms of goal programming). The calculated attainment levels of the two objectives were found to be 66.7 percent for the efficiency-objective and 62.2 percent for the conservation-objective, implying that the both objectives have been achieved in a relatively good balance.

Next let us analyze both alternatives **A – 1** ($\mathbf{G}_2$) and **A – 1** ($\mathbf{G}_3$) where a higher priority has been given to the conservation-objective and to the efficiency-objective, respectively. Then the following will be easily understood (see **Figs. 7 to 10**)

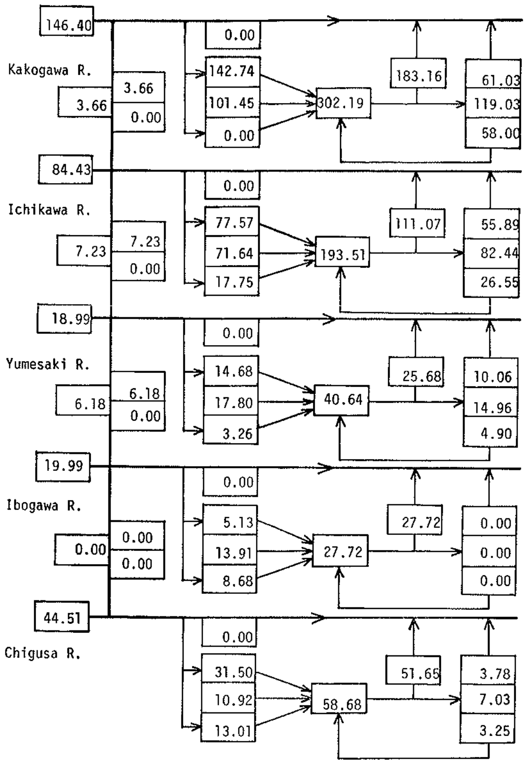


Fig. 7 Computation results (Case A-1 (G_2))
(unit : 10^4 m^3 /day)

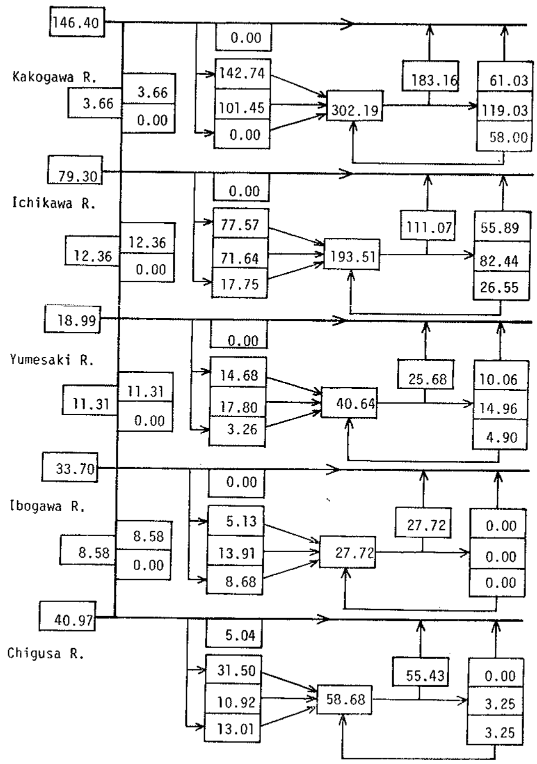


Fig. 8 Computation results (Case A-1 (G_3))
(unit : 10^4 m^3 /day)

(1) There is no basic difference in the water utilization system between alternatives **A–1** ($\mathbf{G}_2$) and **A–1** ($\mathbf{G}_1$), mere difference lying in the amount of streamflow to be diverted from the Kakogawa and Yumesaki River to the Ichikawa River.

(2) Alternative **A–1** ($\mathbf{G}_3$) was found to be relatively similar to alternative **A – 1** (**E**), rather than to alternative **A – 1** ($\mathbf{G}_1$). One essential difference between **A–1** ($\mathbf{G}_3$) and **A–1** (**E**) is that in alternative **A – 1** ($\mathbf{G}_3$) the Yumesaki River

diverts 10,500 m³/day of its streamflow to the Ichikawa River, where in alternative A−1 (C) the Yumesaki receives 9,490 m³/day of streamflow from the Ibogawa River.

(3) So far as the attainment level is concerned, alternatives A−1 (G_2) and A−1 (G_3) can be considered to be lying midway between A−1 (G_1) and A−1 (C), and between A−1 (G_1) and A−1 (E), respectively.

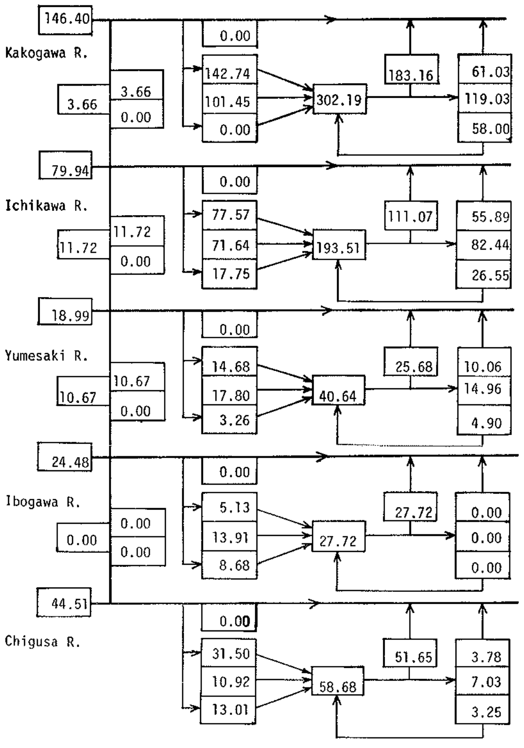


Fig. 9 Computation results (Case A-1) GP)) (unit : 10^4 m^3 /day)

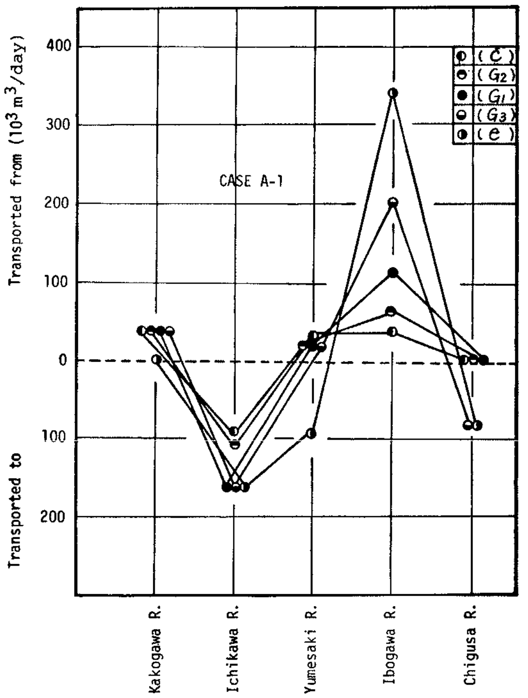


Fig. 10 Streamflow transported to and from each river

6.3 Comparative Study of the Standard Case and the Other Cases

First, let us compare the results of **Cases B−1 and C−1** with those of **Case A−1** (the standard case). We mainly concern ourselves with alternative **B−1** (G_1), **C−1** (G_1) and **A−1** (G_1).

(1) With decrease in water demands, the amount of fresh water to be developed in the Kakogawa River Basin where the demand is greater than those in the rest of the river basins was found to decrease. In contrast the amounts of fresh water to be developed in the Ichikawa and Yumesaki River remain constant.

(2) With decrease in the water demands, the amount of water to be diverted decreases. Much difference was also found in the form of diversion. That is, in **Cases A** and **B**, the Yumesaki River diverts part of its streamflow to the Ibogawa

River, whereas in **Case C** it receives water from the Ibogawa River. This is also true with the Chigusa River which diverts streamflow to the other rivers in **Case A**, whereas it receives water from the Ibogawa River in **Cases B** and **C**.

Next let us compare the results of **Cases A–2** and **A–3** with those of **Case A–1** to examine how a change in the minimum streamflow requirement affects the water utilization system to be implemented. Close scrutiny of the results show :

(1) With decrease in the minimum streamflow requirement, the amount of fresh water to be developed in each of the river basins was found to change a little.

(2) The main difference is that the amount of reclaimed wastewater for industrial use increases as the minimum streamflow requirement decreases. This is mainly because a decrease in the minimum streamflow requirement leads to an increased level of streamflow qualty control, thus leading to an increased amount of reclamation.

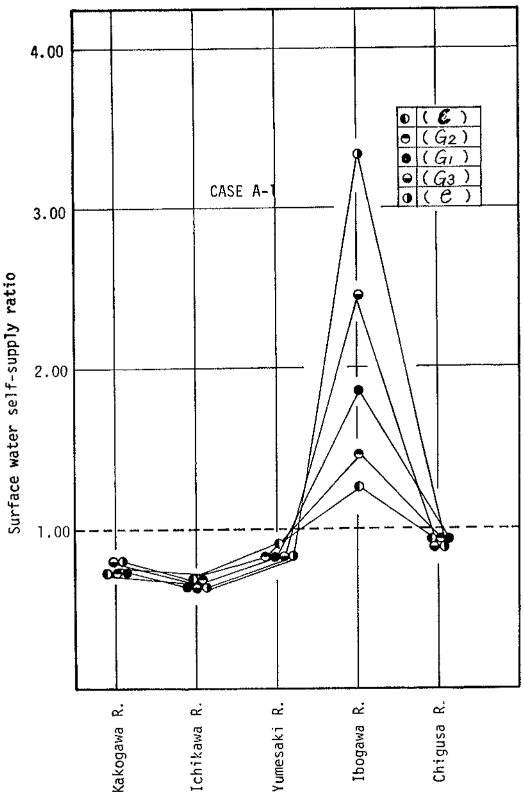
6.4 Analysis of the Results of the Goal Programming Model

Let us examine alternative **A–1 (GP)** against alternative **A–1 (G_1)** which is the initial product derived from the Belenson Method's application to the model. Before going into a comparative study we should observe here that the satisfactory and permissible level for objective j ($j = 1, \cdots, m$) have been set to be equal to f_{jj} and f_{jk_j}, respectively and that the attainment level has been defined in analogy with the Belenson Method. Close scrutinity of the results shows (see **Fig. 11** to **13**) :

(1) There seems to be much similarity found between alternatives **A – 1 (GP)** and **A–1 (G_1)**. The goal programming model leads to a well balanced attainment in each of the objectives–better than the Belenson Method generally in light of balancing the objectives' attainments. This is particularly the case with alternative **A–1 (GP)** which is characterized by the equal attainment of both objectives, 63.7 percent for both the efficiency-and the conservation-objective.

(2) This is considered to have been derived from the fact that the L-type utility function incorporated into the goal programming model is an explicit promoter toward balancing the attainment of each of the objectives, whereas the two-person zero-sum mixed strategy game imcorporated into the determination of the assigned weights in the Belenson Method is considered as an implicit agent of promoting a well-balanced attainment in each of the objectives.

It must also be borne in mind that in obtaining alternative **A–1 (GP)** the satisfactory and the permissible level of each of the objectives are assumed to be equal to f_{jj} and f_{jk_j}, respectively. This assumption is considered to have brought about much similarity in the computation results between the Belenson Method and the goal programming with L-type utility function.



Self-supply ratio for river basin i= $\sum_j x_{ij}/(d^M+d^I)$

Fig. 11 Surface-water vs self-supply ratio calculated for each river

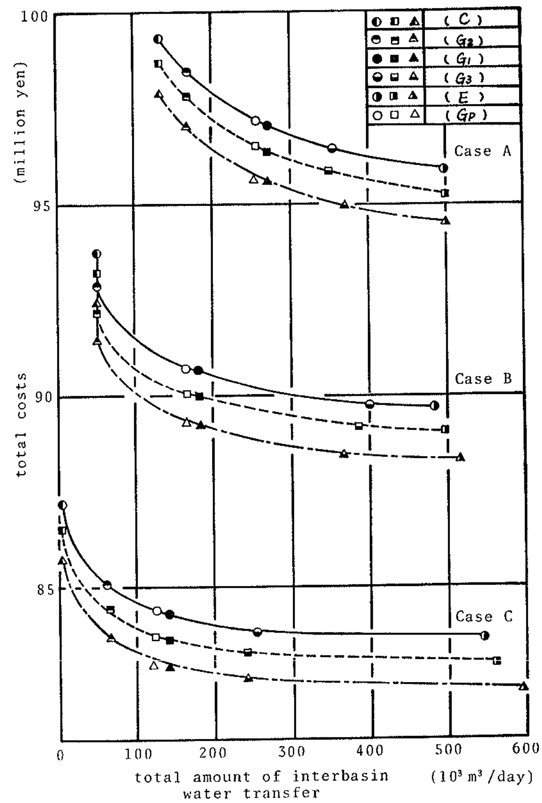
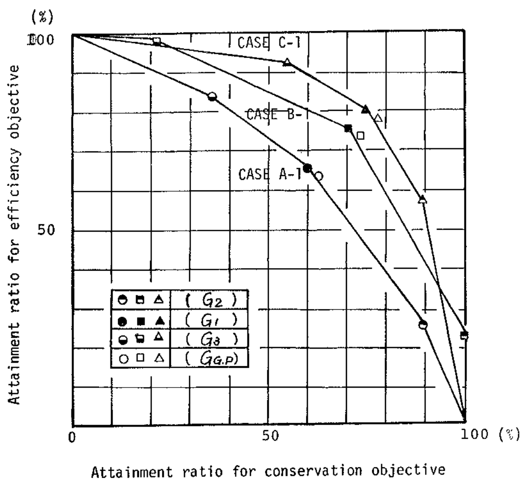


Fig. 12 Total costs vs. total amount of interbasin water transfer



Fig, 13 Trade-off between two objectives

7. CONCLUSION

The intended purpose of this study was to present two possible multi-objective programming approaches to an interbasin water resources allocation problem. That is, the Belenson Method and the Goal Programming have been applied to the defined problem and a comparative analysis of the two methods has been made. A case study has been conducted for the Southern Part of Hyogo Prefecture and close examinations of the computational results obtained from the model application to the region have been made to check the applicability of the methods. Among many findings the following seem to be most significant.

(1) If our main concern is with how to attain a well-balanced attainment of the incorporated objectives, the goal programming with L-type utility function has been found to serve for the purpose in a straight-forward way.

(2) The Belenson Method could also serve for this purpose in an indirect manner, but this seems to be more helpful in providing the decision maker with a set of efficient solutions including such well-balanced alternatives which come out through the articulation of priorities to the set of objectives. This method could be especially encouraging when the decisiion maker is not determined as to whether he should go in favor of ranking each objective as equal importance, but instead he is willing to know what the outcome would be like if the set of objectives were arranged in order of tentative priorities. It must be observed that the Belenson Method works well in this particular case on the assumption that some modification is made on the original method. The difference resides in that it is assumed in the original method that the decision maker be asked to intervene in the solution-finding process to articulate whether the most acceptable alternative has already been found and whether the process should be terminated.

It is claimed that in practical situations we may dispense with this kind of intervening process but provide the decision maker with a set of efficient solutions, leaving it open as to which should be identified as most acceptable. A more reasonable way would be to feed back the analysists (or the planners) with the evaluation of the proposed set of alternatives.

(3) In any event the set of alternatives produced by the Belenson Method can hardly be considered as exhaustive, nor is there any need to cover all possible efficient solutions, because it would no more than confuse the decision maker with a huge number of possible choices.

Though there is still much room for development, we may fairly state that the methods presented here may help the decision-maker systematically assess the promising alternatvies, whereby the order of priority being explicitly articulated for the set of objectives and the resultant planning outputs clearly illustrated for each alternative.

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