# An Interactive Man-Machine Approach to Cost Allocation in Water Resources Development

by

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With the case study of Sweden as a basis, some extended approach has been suggested to the inclusion of dynamic performances into the process of cost allocation in water resources development. A multi-objective programming model has been developed based on the notion of "core" in terms of cooperative games. Some demonstration has been made to suggest the applicability of the model and needed further attempts in line of this paper have been suggested.

## 1. Introduction

In the field of water resources management, there have been mounting concerns about how to reconcile conflicting interests among the different parties involved. Among a variety of conflict problems is the well-known problem : how to split the total costs of a joint project among different users. This problem, which is generally called "cost allocation", is the major concern of this paper.

The water resources field has extensive literature on this theme. Many approaches have been proposed, tested, and modified therein, and some of them appear to have gained extensive publicity and application in this field.

In the following section we shall start with the taxonomy of cost allocation ; thereby referring to the available methods, conventional and newly developed ones, which fall into the different categories of the taxonomy. The discussion will lead to the recent work by Young, Okada and Hashimoto(1) ; its extension being suggested to the dynamic process of allocating costs. Section 3 will discuss a pilot approach to the proposed extension of cost allocation as an inter-active man-machine dialogue system. With the southern part of Sweden called Skane as the study area and based on the above-cited work, a multi-objective programming model will be developed ; followed by the analysis of the computation results derived from the model. In the conclusion assessment will be made of the applicability of the model presented herein and suggestion made on the needed further effort to introduce the suggested approach

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into the field of cost allocation.

## 2. Taxonomy of Cost Allocation

There seems to be a myth in the cost allocation that cost allocation is no more than a financial analysis. For example, James and Lee state[2] that "since cost allocation determines how much each party should pay, it is part of financial analysis. It is never part of economic analysis. Fixed costs which cannot be directly attributed to any project purpose are not marginal and thus have no influence on optimum design (they may affect economic justification) but since they must be paid, financial analysis is required to assign them to someone". This kind of limited definition of cost allocation has prevailed in the field of cost allocation and seems to be accepted as an established axiom. **Table I** lists a set of available cost allocation methods. The most conspicuous and widely used among them is the Separable Cost Remaining Benefit (SCRB) Method.[2][3][4] This method whose origin dates back to the 1950's when it was applied to the T. V. A. projects in the U. S. A., has been further develoed in other countries including Japan to constitute the legal basis of the present cost allocation procedures. Okada[5] has criticized that the SCRB, which represents the conventional cost allocation methods is based also on the limited definition that cost allocation is no more than a financial analysis.

**Table I Cost allocation methods**

| | Amount to Be Allocated | | |
|---|---|---|---|
| | *a* | *b* | *c* |
| Vehicle | Total cost | Direct cost excluded | Separable cost excluded |
| *A* Equal | *Aa* | *Ab* | *Ac* |
| *B* Unit of use | *Ba* | *Bb* | *Bc* |
| *C* Priority of use | *Ca* | *Cb* | *Cc* |
| *D* Net benefit | *Da* | *Db* | *Dc* |
| *E* Alternative cost | *Ea* | *Eb* | *Ec* |
| *F* Smaller of benefit or alternative cost | *Fa* | *Fb* | *Fc* |

To understand how limited the definition is, let us raize the question : Why would the participants agree to stay with the project before they knew their share of costs? This is precisely the question of the participants' incentives. By "incentives" we mean the inducements for prospective uses to take part in a joint project. In light of this consideration Young, Okada and Hashimoto have claimed that a potent way to formulate the incentives of participants is to look at their bargaining structure.

They have identified a set of basic principles that ought to be embodied in cost allocation, have then proceeded to a systematic check of both conventional and game theoretic methods against the basic principles. They proposed that the notion of "core" in terms of cooperative games constitutes the basis of cost allocation and satisfies the principle of "individual" and "group rationality" and "marginality principle". They concluded that the computational methods including the SCRB and some game theoretic methods do not satisfy the core, nor the other principle called "monotonicity principle". They showed that only a couple of lesser known methods from game theory, i. e., the Weak Nucleolus (WN) and the Proportional Nucleolus (PN) Method proved to be more appropriate.

The extent to which cost allocation has implications for incentive analysis may depend largely on the level or stage of planning in which cost allocation is discussed. We may roughly classify the process of planning a water development project into several stages, i. e., motivation, project design and appraisal, project implementation, and operation and maintenance. **Table II** lists the type of analysis required at each stage of planning. This shows that the earlier the stage of planning is, the more important becomes the incentive analysis.

**Table II Planning stage v. s. type of analysis needed**

| Stage of Planning | | Type of Analysis required |
|---|---|---|
| earlier | motivation | incentive analysis, monitoring |
| | investigation | feasibility and incentive analysis, monitoring |
| | project design and appraisal | cost/benefit analysis, impact analysis, incentive analysis, monitoring |
| | project implementation | engineering design, financial analysis, incentive analysis, construction management |
| later ↓ | operation and maintenance | monitoring, financial analysis, technical follow-up, monitoring |

Behind this discussion is aloso the question of whether cost allocation demands a normative approach or an empirical approach. One may argue that cost allocation should be no more than a normative approach. Notably when cost allocation is classified into a financial analysis, it implies that cost allocation is a kind of normative analysis. This type of argument is commonly based on the claim that from the point of view of social fairness and justice and from a managerial standpoint, a procedure must be established which applies to hundreds of similar cost allocation problems.

There is, however, a natural situation in which empirical analysis needs to be done in cost allocation. Suppose there has not yet been any cost allocation procedure established and one desires to pick up those rules or norms which patternize what may turn out to be a normative procedure in the future. Another case may be that although set of norms or principles have been proposed by the project organizer or some of the participants, the participants desire to obtain a deeper understanding of what is implied by the application of those norms to cost allocation. Both situations are similar in that the process of allocating costs is regarded as a kind of learning process by which the participants can come to identify a common set of principles or by which the participants can become more familiar with the preselected principles.

To summarize the above discussion we may state that the conventional methods represented by the SCRB fail to incorporate the function of incentive analysis and empirical analysis. We will develop this discussion more concretely by placing the scope of analysis on the context of the Swedish Case Study as carried out by Young Okada, and Hashimoto.

## 3. Swedish Case Study[1)]

The study area consists of eighteen municipalities in the Skane region of southern Sweden. At present most of the municipal water supply is drawn from three sources : local ground water, and two separate pipeline systems which distribute water from two lakes Vombsjon and Ringsjon. As early as the 1940's, some municipalities in the area realized the possibility of shortages in local water sources and turned their attention to off-site sources. An association called the Sydvatten Company was formed by several of them to plan for long-term water supply and management of the region. In the late 1960's, this group (consisting presently of 12 of the municipalities) began to design a major project to obtain water from a lake outside the region (Lake Bolmen) via an 80 km. tunnel.

The viability of the project depends on how many municipalities will participate in the project, and this in turn is dependent on how much they will be obliged to pay by participating in such a development vis-a-vis the availability and costs of developing their own on-site sources. When they started the discussion of cost allocation, they learned that there had not been no established method available for this type of cost allocation. As a compromise they came to agree that the total joint costs be allocated in proportion to population. Recently this project has been undergoing a period of reconsideration as the actual increase in population and water demand over the past decade has turned out to be short of the original forecasts. Since their cost allocation is based on population, this has prompted disputes over the validity of the employed method.

This problem is presicely what Young, Okada and Hashimoto dealt with. For details of the discussion and the methods developed by them, please see the cited reference.1) As a basis for this study we summarize their work as follows.

1) There are six independent (group of) municipalities, A, H, K, L, M and T as Shown in **Fig. 1**.

2) For each municipality there are basically three alternatives ; (i) going alone by developing its water source on its own ; (ii) staying with the joint enterprise ; and (iii) developing a smaller joint venture by forming a coalition with the other prospective municipalities.

3) **Table III** lists a set of possible alternatives (individual and joint enterprises) against the calculated minimum cost of its implementation. Let this cost, C(S) be called (joint) cost characteristic function defined for coalition S (when the project is undertaken by a single municipality, $\{S\} = A, H, K, L, M$ or $T$ ; which means that the coalition member is single or that there is no coalition in the narrow sense).

4) With this setting in mind they have identified

| Group | Municipalities in the Group |
|---|---|
| A | Ängelholm, Höganäs, Klippan, Åstorp, Bjuv |
| H | Helsingborg, Landskrona, Svalöv, Eslöv |
| K | Kävlinge, Lomma |
| L | Lund |
| M | Malmö, Burlöv, Staffanstorp |
| T | Trelleborg, Vellinge, Svedala |

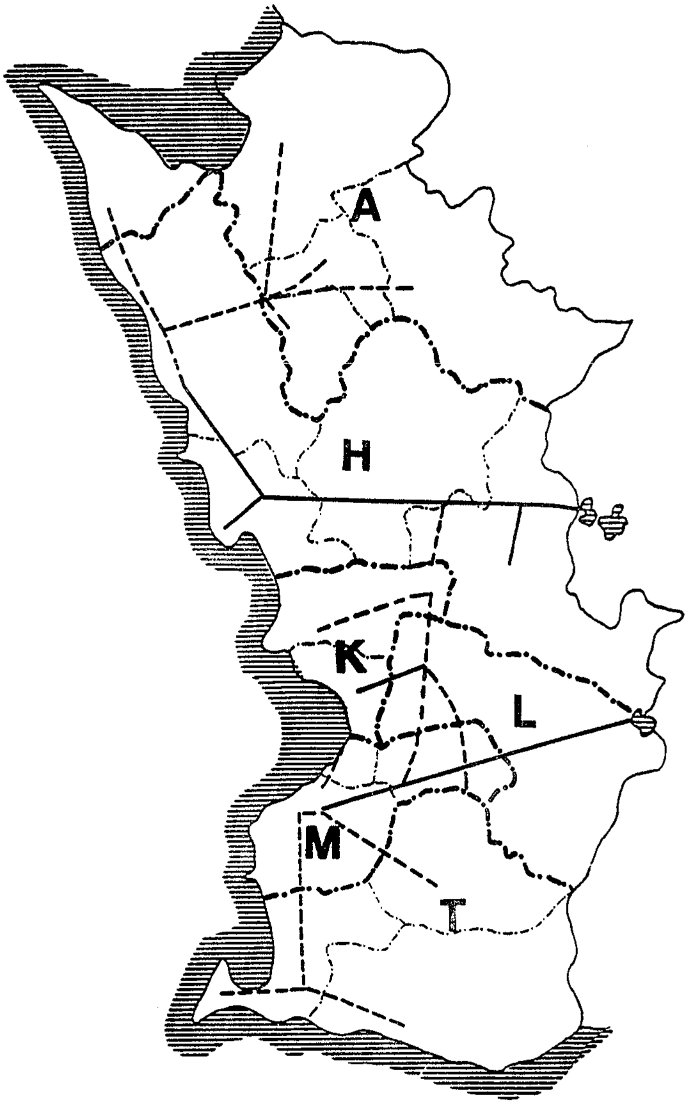

**Fig. 1.** Study area and grouping of 18 municipalities.

**Table III** **Coalition patterns and joint cost characteristic functions calculated for the Swedish case study (Millions of Swedish crowns).**

| | | | | | |
|---|---|---|---|---|---|
| A | 21.95 | AHK | 40.74 | AHKL | 48.95 |
| H | 17.08 | AHL | 43.22 | AHKM | 60.25 |
| K | 10.91 | AH,M | 55.50 | AHK,T | 62.72 |
| L | 15.88 | AH,T | 56.67 | AHL,M | 64.03 |
| M | 20.81 | A,K,L | 48.74 | AHL,T | 65.20 |
| T | 21.98 | A,KM | 53.40 | AH,MT | 74.10 |
| | | A,K,T | 54.85 | A,K,LM | 63.96 |
| AH | 34.69 | A,LM | 53.05 | A,K,L,T | 70.72 |
| A,K | 32.86 | A,L,T | 59.81 | A,LMT | 73.41 |
| A,L | 37.83 | A,MT | 61.36 | HKL,M | 48.07 |
| A,M | 42.76 | HKL | 27.26 | HKL,T | 49.24 |
| A,T | 43.93 | HKM | 42.55 | HKMT | 59.35 |
| HK | 22.96 | HK,T | 44.94 | HLMT | 64.41 |
| HL | 25.00 | HL,M | 45.81 | KLMT | 56.61 |
| H,M | 37.89 | HL,T | 46.98 | A,K,MT | 72.27 |
| H,T | 39.06 | H,MT | 56.49 | AHKLM | 69.76 |
| K,L | 26.79 | K,LM | 42.01 | AHKMT | 77.42 |
| KM | 31.45 | K,L,T | 48.77 | AHLMT | 83.00 |
| K,T | 32.89 | K,MT | 50.32 | AHKL,T | 70.93 |
| LM | 31.10 | LMT | 51.46 | AKLMT | 73.97 |
| L,T | 37.86 | | | HKLMT | 66.46 |
| MT | 39.41 | | | | |
| | | | | AHKLMT | 83.82 |

a set of principles to base the method of cost allocation. They are : (i) individual and group rationality which refer to the qualification that no participant or group of participants would be induced to stay with the joint enterprise if he or those who could contemplate the formation of their own coalition, were asked to pay more than the cost of his indivudual project or of their joint project ; (ii) marginality principle which means that every collection of users should be charged at least as much as the additional cost of serving them ; (iii) the principle of monotonicity which says that if costs turn out to be higher than expected then no participant's allocation should go down, and vice versa. It has been shown that the condition which satisfies both (i) and (ii) is what has been known as the notion of "core".

5) In order to single out a solution among those which satisfy the core, the Weak Nucleolus (WN) and the Proportional Nucleolus (PN) have been developed. It has been theoretically proved and illustrated by the case studv that these two methods are more reasonable than the conventional methods and some other game-theoretic mothods. For reference the computation results are shown in **Table IV** for the selected different methods.

**Table IV Computetion results for the selected methods**

| Method | A | H | K | L | M | T |
|---|---|---|---|---|---|---|
| Proportional to Population | 10.13 | 21.00 | 3.19 | 8.22 | 34.22 | 7.07 |
| Proportional to Demand | 13.33 | 16·32 | 7.43 | 7.00 | 29.04 | 10.69 |
| SCRB | 19.54 | 13.28 | 5.62 | 10.90 | 16.66 | 17.82 |
| Shapley Value | 20.01 | 10.71 | 6.61 | 10.37 | 16.94 | 19.18 |
| Nucleolus | 20.35 | 12.06 | 5.00 | 8.61 | 18.32 | 19.49 |
| Proportional Nucleolus | 19.81 | 12.57 | 4.35 | 9.25 | 18.34 | 19.47 |
| Weak Nucleolus | 20.03 | 12.52 | 3.94 | 9.07 | 18.54 | 19.71 |

Based on the findings of the study the prospective participants were asked to assess its conclusion. In general the study interested them very much and provided them with a bisis for the improvement of their method. It has been suggested, however, that they would like the poroposed game theoretic approach to include the dynamic aspect of cost allocation, because the actual process of cost allocation develops over time on a trial-and-error basis when there is no agreement on what particular method should be employed.

This suggestion has motivated the intiation of two studies. One is the study by Stahl and another is this paper. Stahl proposed a gaming simulation approach to the above-cited cost allocation. He invited the water managers from the respective municipalities to an experiment with a set of instructions prescribed for the cost allocation game. Based on the experimental result he suggested that the particpants' behavioral pattern proved to be well explained by what is implied by the Shapley Value which is another type of game theoretic approach that is not based on the notion of "core".[6)]

This paper presents multi-objective programmimg approach based on the cost. We shall discuss this approach in the next section.

## 4. A Core-based Cost Allocation Game—A multi-objective programming approach

### 4.1 Assumption

(1) The game is to be based on the notion of core as the fundamental set of constraints on cost allocation.

(2) It is known that the core exists for a given cost allocation problem. Players (participants) are requested to reach a compromise solution which they can select from the set of alternatives satisfying the core.

(3) Each individual player bears his own goal in mind and wishes that his goal will be attained as highly as possible. He is asked to prescribe for his goal the satisfac-

tory and the permissible level of attainment (accordingly an allowable band of goal attainment).

(4) Each player's goal is defined as minimizing the cost to be allocated to himself.

(5) The permissible level for each player's goal is identified with the principle of "individual rationality". Let $\underline{g}_i$ represent the permissible level for the goal of player $i$. Then $\underline{g}_i$ is defined as :

$$\underline{g}_i = c\ (\{i\})\quad (i = 1, \cdots n\ ;\ n = 6 \text{ for the case study}). \cdots\cdots\cdots\cdots (1.1)$$

(6) There are several variants of formulation to be developed, depending on how one specifies the satisfactory level for his goal. Two of those candidates are sugges ted :

**a.** We take the marginal cost computed for each individual as the satisfactory level for his goal. That is, for player $i$ :

$$\bar{g}_i = c\ (N) - c\ (N - \{i\})\quad (i = 1, \cdots, n), \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1,2)$$

where $\bar{g}_i$ represents the satisfactory level for the goal of player $i$ ; and $N$ stands for the grand coalition which is the largest joint venture, the cost allocation of which is of our concern.

**b.** Alternately we set as the satisfactory level the minimum of the marginal costs computable for all subsets of feasible coalition patterns. That is, for player $i$ :

$$\bar{g}_i = \min_{S<N} \{C\ (S + \{i\}) - C\ (S)\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1,3)$$

(7) Since their goals would conflict if all the goals were attained to maximum, the game is formulated as a multi-objective programming problem.

(8) There are a number of approaches available for both formulating and solving this type of problem. As one promising approach let us take a goal programming approach with the L-type utility function.7)8) We also assume that all the players have agreed that the goals should be well balanced in attainment. By "well-balanced" we mean that the extent to which the achievement of one's objective is remote from his satisfactory level needs to be as close as possible to the extent to which the achievement of the other's objective is remote from his (the other's) satisfactory level.

## 4.2 Model formulation

Based on the above assumptions let us formulate the problem by applying the goal programming based on the L-type

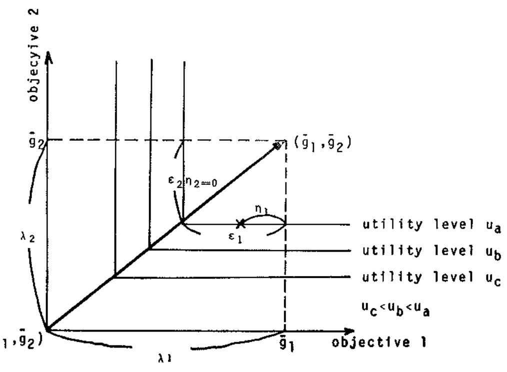

Fig 2

Fig. 2. Goal programming withL-type utility function.

utility function between goals (see **Fig.** 2)

**a. Objective function**

$Min\ \varepsilon_{i_0}$ ($i_0$ being arbitrary $i$ ; $i = 1, \cdots, n$)$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$(1.4)

**b. Basic constraints**

group rationality : $\sum_{i \in S} x_i \leqq c(s)$ $(S<N,\ S = \{i\},\ i = 1, \cdots, n)\cdots\cdots$(1.5)

total allocation : $\sum_{i \in N} x_i = c(N)$ $(i = 1, \cdots, n)\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$(1.6)

**c. Goal Constraints**

By taking the individual rationality as the permissible level we get :

$x_i \leqq c(\{i\}) = g_i$ $(i = 1, \cdots, n)$ $\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$(1.7)

Let us take (i. 2) as the satisfactory level. Then we get :

$X_i - \varepsilon_i + \eta_i = \overline{g}_i$ $(i = 1, \cdots, n),\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$(1.8)

where $\varepsilon_i$ and $\eta_i$ are deviational variables as illustated in **Fig. 2** $(\varepsilon_i\ ,\ \eta_i\ \geqq 0)$ and the follwowing relation holds between deviational variables :

$\varepsilon_1 / \lambda_1 = \varepsilon_i / \lambda_i$ $(i = 2, \cdots\cdots, n)\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$(1.9)

$\lambda_i = | \overline{g}_i - \underline{g}_i |$ $(i = 1, \cdots\cdots, n)\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$(1.10)

### 4.3 Operation Strategy

The model should be operated intuitively so that continued interactive dialogues with the participants can be maintained and the computation results from one iteration may be fed back into the following iteration. For this purpose we need to bring in another rule.

One basic example for this kind of rule is suggested :

(1) After each iteration the best treated participant is identified by some criterion set a priori. A candidate criterion may be the attainment ratio as defined for participant $i$ by : $| \underline{g}_i - x_i | / \lambda_i$ .

(2) The participant, say $i^*$ who has been identified as the best treated one will be asked by the rest of the participants to lower his satisfactory level for his own, $g_i^*$ to $g_i^{*\prime}$ where $g_i^* < g_i^{*\prime}$. It may also be the case that some or all of the other members excluding the one best treated may enter into negotiation with the others to form a couter-coalition against the one best treated ; thus indirectly pressing this participant into giving up part of the achievement he has enjoyed in the current cost allocation pattern. We will leave this mechanism open to the participants.

(3) For generality let us distinguish between two kinds of attainment ratio definable for participant $i$ at iteration stage $k$, i. e., the absolute attainment ratio (AAR) and the relative attainment ratio (RAR). They are defined as :

$$AAR_i = | \underline{g}_i^{(1)} - x_i^{(k)} | / \lambda^{(1)} \cdots\cdots (1.11)$$

$$RAR_i = | \underline{g}_i^{(k)} - x_i^{(k)} | / \lambda^{(k)} \cdots\cdots (1.12)$$

A set of scenarios presupposed is listed in **Table V.**

**Table V Operation strategy for the game**

| | | A | H | K | L | M | T |
|---|---|---|---|---|---|---|---|
| 1 | $\overline{g}$ | 17.36 | 9.85 | 0.82 | 6.4 | 12.89 | 14.06 |
| | $\underline{g}$ | 21.95 | 17.08 | 10.91 | 15.88 | 20.81 | 21.98 |
| | $\lambda$ | 4.59 | 7.23 | 10.09 | 9.48 | 7.92 | 7.92 |
| | additional condition | none | none | none | none | none | none |
| 2 | $\overline{g}$ | 13.89 | — | — | — | 10.31 | 11.25 |
| | $\underline{g}$ | — | — | — | — | — | — |
| | $\lambda$ | 8.06 | — | — | — | — | — |
| | additional condition | — | — | — | — | — | — |
| 3 | $\overline{g}$ | — | — | — | 5.12 | — | — |
| | $\underline{g}$ | — | — | — | — | — | — |
| | $\lambda$ | — | — | — | 10.76 | — | — |
| | additional condition | — | — | — | — | — | — |
| 4 | $\overline{g}$ | — | eliminated | — | — | — | — |
| | $\underline{g}$ | — | eliminated | — | — | — | — |
| | $\lambda$ | — | eliminated | — | — | — | — |
| | additional condition | — | $x_H = 10.0$ | — | — | — | — |
| 5 | $\overline{g}$ | — | — | — | — | — | — |
| | $\underline{g}$ | — | — | — | — | — | — |
| | $\lambda$ | — | — | — | — | — | — |
| | additional condition | — | $x_H + x_K + x_L = 25.0$ | | | | |
| 6 | $\overline{g}$ | — | — | — | eliminated | — | — |
| | $\underline{g}$ | — | — | — | eliminated | — | — |
| | $\lambda$ | — | — | — | eliminated | — | — |
| | | — | — | — | $x_L = 10.0$ | — | — |

— : No change to the preceding problem

### 4.4 Demonstration

To put the discussin on the context of the Swedish case study, let us base our game on the same data as shown in **Table III.** It is noted that the implication and role of this type of game should be empirically demonstrated by applying the technique to a forum where those people who really represent the interests of six municipalities, **A**, **H**, **K**, **M** and **T** will be asked to play the game. Practically, however, the typical patterns may not be so many. Some of them may even be predictable at the risk of oversimplifying what otherwise would occur in allocating the costs. It is hoped that to pick up one plausible example may help expose the implication and role of the game. In light of this consideration a demonstration will be made on the basis of a preassumed set of senarios (see **Table VI.**)

**Table VI Results of the game**

| | Player | A | H | K | L | M | T |
|---|---|---|---|---|---|---|---|
| 1 | $x$ | 20.112 | 9.850 | 5.326 | 12.084 | 17.639 | 18.809 |
| | $\xi$ | 2.752 | 5.211 | 6.050 | 5.684 | 4.749 | 4.749 |
| | $\eta$ | 0.0 | 5.211 | 1.544 | 0.0 | 0.0 | 0.0 |
| | RAR | 0.40 | 1.0 | 0.57 | 0.40 | 0.40 | 0.40 |
| | AAR | — | — | — | — | — | — |
| 2 | $x$ | 19.699 ↓ | 9.850→ | 4.188 ↓ | 13.222 ↑ | 17.878 ↑ | 18.983 ↑ |
| | $\xi$ | 5.809 | 5.211 | 7.272 | 6.833 | 7.568 | 7.733 |
| | $\eta$ | 0.0 | 5.211 | 0.0 | 0.0 | 0.0 | 0.0 |
| | RAR | 0.28 | 1.0 | 0.67 | 0.28 | 0.28 | 0.28 |
| | AAR | 0.49 | 1.0 | 0.67 | 0.28 | 0.37 | 0.39 |
| 3 | $x$ | 19.699→ | 9.850→ | 4.535 ↑ | 12.875 ↓ | 17.878→ | 18.983→ |
| | $\xi$ | 5.809 | 5.211 | 7.272 | 7.755 | 7.568 | 7.733 |
| | $\eta$ | 0.0 | 5.211 | 3.557 | 0.0 | 0.0 | 0.0 |
| | RAR | 0.28 | 1.0 | 0.63 | 0.28 | 0.28 | 0.28 |
| | AAR | 0.49 | 1.0 | 0.63 | 0.32 | 0.39 | 0.38 |
| 4 | $x$ | 19.699→ | 10.000 ↑ | 4.385 ↓ | 12.875→ | 17.878→ | 18.983→ |
| | $\zeta$ | 5.809 | — | 7.272 | 7.755 | 7.568 | 7.733 |
| | $\eta$ | 0.0 | — | 3.707 | 0.0 | 0.0 | 0.0 |
| | RAR | 0.28 | — | 0.61 | 0.28 | 0.28 | 0.28 |
| | AAR | 0.49 | 0.98 | 0.61 | 0.32 | 0.39 | 0.38 |
| 5 | $x$ | 20.321 ↑ | 10.000→ | 2.588 ↓ | 12.412 ↓ | 18.688 ↑ | 19.811 ↑ |
| | $\xi$ | 6.431 | — | 8.056 | 8.585 | 8.378 | 8.561 |
| | $\eta$ | 0.0 | — | 6.283 | 1.293 | 0.0 | 0.0 |
| | RAR | 0.20 | — | 0.82 | 0.32 | 0.20 | 0.20 |
| | AAR | 0.35 | 0.98 | 0.82 | 0.37 | 0.27 | 0.27 |
| 6 | $x$ | 20.321→ | 10.000→ | 4.000 ↑ | 11.000 ↓ | 18.688→ | 19.811→ |
| | $\xi$ | 6.431 | — | 7.051 | — | 8.378 | 8.561 |
| | $\eta$ | 0.0 | — | 3.871 | — | 0.0 | 0.0 |
| | RAR | 0.20 | — | 0.68 | — | 0.20 | 0.20 |
| | AAR | 0.35 | 0.98 | 0.68 | 0.51 | 0.27 | 0.27 |

(1) The game is initiated by the specification of both satisfactory and permissible level for the individual participant's goal. The results of the computations by use of our model are listed in **Table V**.

(2) stage 1 : We get the following cost allocation.

$$x_A = 20.112,\ x_H = 9.850,\ x_K = 5.326,\ x_L = 12.084,$$
$$x_M = 17.639,\ x_T = 18.809.$$

We note that :

$$x_H + x_K + x_L = 27.26 = c\ (H, K, L),$$
$$x_A + x_M + x_T = 56.56 = c(A, H, KL, MT) - c\ (H, K, L).$$

This implies that **A**, **M** and **T** have allied to keep their total share to the minimum by demanding **H**, **K** and **L** to share s total cost of 27.26 which is the maximum **H**, **K** and **L** could share. It should be noted, however, that the grand coalition has not yet been totally broken down but it is still maintained in the sense that **L** functions as an interface between (**H**, **K**) and (**A**, **M**, **T**). That is, **L** enjoys the benefit of the property imbedded in the model that the attainment of **L**' s goal should be balanced against the attainment of the others'. So do the group (**A**, **M**, **T**). It is observed that the attainment ratio for **L** equates those for **A**, **M** and **T** (the ratio being 0.40), whereas those for **H** and **K** are 1.0 and 0.57, respectively.

Given this result, a natural reaction by the group (**A**, **M**, **T**) might be such that they argue that **H** should share more than it currently does. Without directly forcing **H** to increase its share **A**, **M** and **T** may attempt to pressure **H** indirectly by demanding higher satisfactory levels for themselves. We assume that the satisfactory levels for **A**, **M** and **T** have been raised while leaving the rest of conditions unchanged as shown in **Table IV**.

(3) stage 2 : The result is that **A** enjoys a higher attainment in goal, whereas **M**, **T** and **L** get lower attainments. (Note that this is the case if measured in terms of *AAR*. If measured with *RAB*, the attainment ratio for **A** equates those for **M**, **T** and **L**).

Notably **K** turn out to share a less cost than it did in the previous stage, although **K** did not raise its satisfactory level. This has been caused by the fact that **L** has still kept in touch with the group (**A**, **M**, **T**) ; thereby implicitly agreeing on the mechanism of balancing the attainment of his goal against those of **A**, **M** and **T**. It should be observed also that **H** shares as he did before.

Let us assume that **L** still sticks to maintaining its contact with (**A**, **M**, **T**) but wants to do so only by bargaining with (*A*, **M**, **T**) to increase his satisfactory level.

(4) stage 3 : This results in an increased attainment for **L** and a decreased attainment for **K** (**K** being compensating as much as **L** gives up to share), while no change whatsoever in the attainments for **A**, **M** and **T**. Note also that there is still no

change in the share of **H**.

It will not be later than in this stage that **K** and **L** will have become aware of the fact that a higher attainment will be achieved only through bargaining jointly with **H** in order to press **H** to share more. It is assumed that **K** and **L** have succeeded in persuading **H** to share as much as 10.0 (not more than that ; nor less than that).

(5) stage 4 : This leads the participants to the situation in which only **K** enjoys the benefit of decrease in share, whereas **A**, **M**, **T** and even **L** have to share as much as they had to in the previous stage. **A** natural reaction from **K** might be that **L** wants to force **K** to share more. **K** may, however, argue that since he has known that **K** could share less (if the rest of participants except **L** accepted the cost allocation of stage 2), **L** would have to share more.

**L** now knows that **L** should get **H** as well as **K** on his side in order to bargain with the group (**A**, **M**, **T**), arguing that the group (**H**, **K**, **L**) are llegitimately unfavored by the enforced total share of 27.26. Suppose this bangaining turns out to be successful. Then, (H, K, L,) are allocated as much as 25.00 in total.

(6) stage 5 ; The result is thak **K** and **L** enjoy a benefit of decrease in share, while **A**, **M** and **T** have to share more. After this **L** may start bargaining again with **K** by saying that **L** should share as much as 4.000 which is still less than he would have to share at best (by referring to the result of stage 2).

(7) stage 6 ; If **K** yields to **L** to accept this argument, **L** would lower its burden by 2, while **K** would have to increase its burden by the the same amount.

(8) The process further goes on until the participants come to agree on a unique solution. In due course of time, however, it may be likely that the scope of cost allocation is narrowed and screened so the process may not be repeated endlessly. As has been demonstrated in the preceding discussion the participants have been learning the implications of what they a priori structured in their cost allocation procedure. They have also learned about the resultant outputs (cost allocation patterns) to be derived from feeding the input conditions prescribed by them into the core-based model. In this context it may also help the participants become aware of the characteristics of the notion of core in a clearly specified scope. They may even learn that to break the endless process they will need to add some norms or principles to the notion of core, as has been suggested by Young, Okada and Hashimoto.1)

## 5. Conclusion

With the case study of Sweden as a basis, some extended approach has been suggested to the inclusion of dynamic performances into the process of cost allocation. A multi-objective programming model has been developed based on the notion of core. Some demonstration has been made to suggest the needed further attempt to develop this type of model. There will be natural situations which demand of the proposed model. The Swedish case which motivated this study has proved to be an excellent example. More generally the model of this kind may find wide application in diverse situations where participants desire to obtain a deeper understanding of the implications of what they have tentatively agreed to be the basis of cost allocation. In this sense the model, with further development and experiment, will provide the participants in a joint enterprise with a potent tool to learn the mechanism of cost allocation. It is also hoped that it may help them become aware of the need for the incorporation of some other norms or principles ; thus developing an excellent interface with the work done by Young, Okada and Hashimoto.

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