Carrier Wave on Transverse Field Type Electron Beam in Solids*

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The kinetic power theorem for transverse field type electron beam in semiconductors is reviewed on the basis of the first order linearized set of Maxwell's equations and the equations of motion whch include the effects of collisions and diffusion. Kinetic power flow is given in terms of the equivalent kinetic voltages due to longitudinal and transverse velocity modulation, and equivalent current densities.

Dispersion relation is also found for this type of electron beam in a thin semiconductor slab.

§ 1. Introduction

In recent years, several experiments and theories have been reported of a solid state travelling-wave amplifier of semiconductors. These treat the same arrangements as the longitudinal electron beam of the vaccum travelling wave tube.[1-5] On the other hand, the theories and experiments for the same operation principles as transverse field type have little been presented. Lately, a theoretical treatment for the case of a transverse magnetic field has been shown by Kino.[6] However, the temperature effect is not taken into account.

In this paper, the conservation principle and the dispersion relation including the effect are derived. Only generalized and fundamental relations are found. Feasibility for such devices is not discussed.

§ 2. The Fundamental Equations and the Complex Poynting Theorem

The geometric arrangement of the transverse field type electron beam is shown in Fig. 1. We assume that all ac quantities vary as exp(jωt). The linearized equations of motion are given by[7-9]

$$m^* \left\{ (j\omega+\nu)\, v_{1y} + v_0 \frac{\partial}{\partial z} v_{1y} + v_{1z} \frac{\partial v_0}{\partial y} \right\} = qE_{1y} + v_{1z}\, (\nabla \times \boldsymbol{p}_0)_x - \frac{q}{\rho_0} \frac{\partial p_1}{\partial y}, \quad (2.1)$$

$$m^* \left\{ (j\omega+\nu)\, v_{1z} + v_0 \frac{\partial}{\partial z} v_{1z} \right\} = qE_{1z} - v_{1y}\, (\nabla \times \boldsymbol{p}_0)_x - \frac{q}{\rho_0} \frac{\partial p_1}{\partial z},$$

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and

$$(\nabla \times \boldsymbol{p}_0)_x = qB_0 + m^* \frac{\partial v_0}{\partial y} , \tag{2.2}$$

where $\boldsymbol{p}_0$ is the dc total momentum defined by $m\mathbf{v}_0 + q\mathbf{A}_0$ [with $\mathbf{A}_0$ the vector potential. ω, ν, q and m^* represent the angular frequency, the collision frequency, the carrier charge and the effective mass. Except where otherwise stated subscript 0 denotes a dc component, subscript 1 an ac component. Then, p_1 denotes the ac component of pressure and ρ_0 the charge density. E_{1x}, E_{1y} are the electric fields and v_{1x}, v_{1y} the ac velocities. The equivalent current densities K_{1y} and K_{1z} are

$$K_{1y} \equiv j\omega\rho_0 y_1 ,$$
$$K_{1z} \equiv J_{1z} + \frac{\partial}{\partial y} (\rho_0 v_0 y_1) , \tag{2.3}$$

where J_{1z} is the current density. y_1 represent the ac displacement in the y-direction.

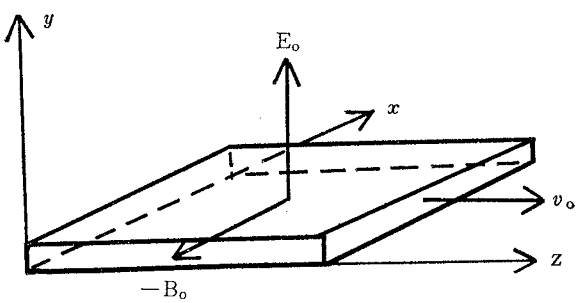

Fig. 1 The geometric arrangement of the electron beam. Dc electric field is applied in the y-direction and dc magnetic field in the x-direction. The electron beam moves in the z-direction with a constant drift velocity v₀. The electron beam has a thickness d and a unit length in the x-direction.

The equivalent current density K_{1y} is connected with v_{1y} :

$$(j\omega + v_0 \frac{\partial}{\partial z}) K_{1y} = j\omega\rho_0 v_{1y} \tag{2.4}$$

The continuity equation reads

$$j\omega\rho_1 + \frac{\partial J_{1y}}{\partial y} + \frac{\partial J_{1z}}{\partial z} = 0 \tag{2.5}$$

The current densities are

$$J_{1z} = \rho_0 v_{1z} + \rho_1 v_0 , \tag{2.6}$$

$$J_{1y} = \rho_0 v_{1y} = \rho_0 (j\omega + v_0 \frac{\partial}{\partial z}) y_1 .$$

Using eqs. (2.4) through (2.6), K_{1z} is related to v_{1z} and K_{1y}

$$(j\omega + v_0 \frac{\partial}{\partial z}) K_{1z} = j\omega\rho_0 v_{1z} + K_{1y} \frac{\partial v_0}{\partial y} . \tag{2.7}$$

The complex Poynting theorem for the transverse field type of electron beam is given by[7]

$$\int_{z1}^{z2} dz \left\{ S_y(a, z) - S_y(-a, z) \right\} + \int_{-a}^{a} dy \left\{ S_z(y, z_1) - S_z(y, z_2) \right\}$$
$$= -\frac{j\omega}{2} \int_{z1}^{z_2} dz \int_{-a}^{a} dy \, (\mu \mid H_1 \mid^2 - \varepsilon \mid E_1 \mid^2)$$
$$- \frac{1}{2} \int_{z_1}^{z_2} dz \int_{-a}^{a} dy \left\{ E_{1y} K^*_{1y} + E_{1z} K^*_{1z} + \frac{\partial}{\partial z} (\rho_0 v_0 E_{1y} y^*_1) \right\} , \tag{2.8}$$

where S_y and S_z denote the components of the vector $\boldsymbol{S}$ (y, z) given by

$$\boldsymbol{S}(y, z) = \frac{1}{2} \boldsymbol{E}_1 (y, z) \times \boldsymbol{H}^*_1 (y, z) . \qquad (2.9)$$

Integration is taken over the volume V as shown in Fig. 2.

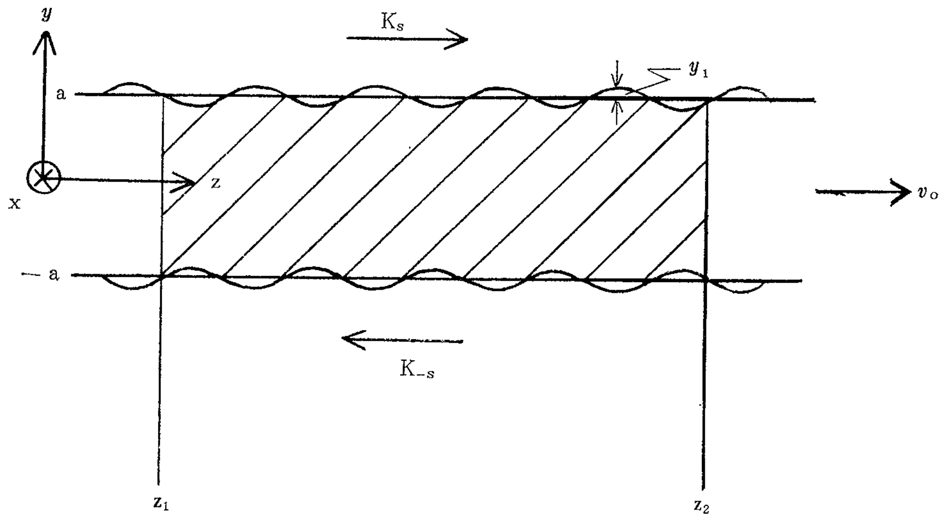

Fig. 2 The electron beam bulges and contracts, following the displacement y_1 (y) in the y-direction. The surface currents (K_s , K_{-s}) are introduced and integration is taken over the dashed region.

We put the last term on the right-hand side of eq. (2.8) as G, coming from existence of the carrier motion. The term $E_{1y} K_{1y}^*$ may be obtained from dot-multiplying eq. (2.1) by K_{1y}^* and using eq. (2.4). The term $E_{1z} K_{1z}^*$ is obtained in a similar manner. Substitution of these into eq. (2.8) gives

$$G = \frac{j\omega\rho_0}{2} \int_{z_1}^{z_2} dz \int_{-a}^{a} dy \left\{ \frac{m^*}{q} (v_{1y} v^*_{1y} + v_{1z} a^*_{1z}) \right.$$

$$\left. + \frac{1}{q} (\nabla \times \boldsymbol{p}_0)_x \left(y^*_1 v_{1z} + y_1 v_{1z}^* + y_1 y^*_1 \frac{\partial v_0}{\partial v} \right) - \frac{m^*}{q} \frac{v_{th}^2}{\rho_0^{\,2}} \frac{\rho_1 \rho_1^*}{\rho_0^{\,2}} \right\}$$

$$+ \frac{1}{2} \int_{z_1}^{z_2} dz \int_{-a}^{a} dy\, v \left\{ \frac{m^*}{q} (K_{1y}^* v_{1y} + K_{1z}^* v_{1z}) \right\}$$

$$+ \frac{1}{2} \int_{z_1}^{z_2} dz \int_{-a}^{a} dy \frac{\partial}{\partial z} \left\{ \frac{m^*}{q} v_0 v_{1z} K_{1z}^* + \frac{m^*}{q} v_0 v_{1y} K_{1y}^* \right.$$

$$\left. + \frac{1}{q} (\nabla \times \boldsymbol{p}_0)_x v_0 K_{1z}^* y_1 + \rho_0 v_0 E_{1y} y_1^* + \frac{\rho_1}{\rho_0} \frac{kT}{q} K_{1z}^* \right\}$$

$$+ \frac{1}{2} \int_{z_1}^{z_2} dz \int_{-a}^{a} dy \frac{\partial}{\partial y} \left(\frac{\rho_1}{\rho_0} \frac{kT}{q} K_{1y}^* \right) \qquad (2.10)$$

where $v^2{}_{th}$, T and k are the r. m. s. value of the thermal velocity, the room temperature and the Boltzmann constant. We define each term of eq. (2.10) as H_1, H_2, H_3 and H_4, respectively. Then the following interpretations are put on each term.[7),10)]

(1) H_1 is the time averaged kinetic energy of the electron beam. The third is a newly added term implying the energy due to pressure.

(2) H_2 is a newly added term representing the dissipation power, which may be rewritten as,

$$H_2 = \frac{1}{2}\int_{z_1}^{z_2} dz \int_{-a}^{a} \frac{1}{l}\left\{V_{vy} K_{1y}^* + V_{vz} K_{1z}^*\right\} \tag{2.11}$$

where l is the mean free path.

(3) H_3 is the kinetic power flow along the electron beam and may be written in terms of the equivalent kinetic voltages,

$$H_3 = \frac{1}{2}\int_{-a}^{a} dy \left\{V_{vy} K_{1y}^* + V_{vz} K_{1z}^* + \frac{1}{q}(\nabla \times \boldsymbol{p}_0)_x v_0 y_1 K_{1z}^* + \rho_0 v_0 y_1{}^* E_{1y} + V_p K_{1z}^*\right\}\Bigg|_{z_1}^{z_2} \tag{2.12}$$

The fifth coming from pressure is a newly added term.

(4) H_4, which is newly added, represents the power flow due to pressure in the y-direction. This may be written with V_p

$$H_4 = \frac{1}{2}\int_{z_1}^{z_2} dz \int_{-a}^{a} dy \frac{\partial}{\partial y}(V_p K_{1y}^*) \tag{2.13}$$

The kinetic voltages in the above equations are defined as

V_{vy} = the kientic voltage in the y-direction = $m^* v_0 v_{1y}/q$

V_{vz} = the kinetic voltage in the z-direction = $m^* v_0 v_{1z}/q$

V_p = the kinetic voltage of density modulation = $(\rho_1/\rho_0)(kT/q)$

Thus the complex Poynting theorem (2.8) is finally brought in the form,

$$R_e\int_{z_1}^{z_2} dz \left\{S_y(a, z) - S_y(-a, z)\right\} + R_e\int_{-a}^{a} dy \left\{S_z(y, z_2) - S_z(y, z_1)\right\}$$
$$+ R_e\int_{z1}^{z2} dz \left\{S_{by}(a, z) - S_{by}(-a, z)\right\} + R_e\int_{-a}^{a} dy \left\{S_{bz}(y, z_2) - S_{bz}(y, z_1)\right\}$$
$$= -\frac{1}{2} R_e \int_{z1}^{z2} dz \int_{-a}^{a} dy \frac{1}{l}(V_{vz} K_{1z}^* + V_{vy} K_{1y}^*) \tag{2.14}$$

where

$$S_{bz}(y, z) = \frac{1}{2}\left\{V_{vz} K_{1z}^* + V_{vy} K_{1y}^* + V_p K_{1z}^* + \frac{1}{q}(\nabla \times \boldsymbol{p}_0)_x v_0 y_1 K_{1z}^* + E_{1y} \rho_0 v_0 y_1{}^*\right\} \tag{2.15}$$

and

$$S_{by}\ (y,\ z)\ =\ \frac{1}{2}\ V_p\, K_{1y}^{*} \tag{2.16}$$

The first term on the left-hand side of eq. (2.14) is the electromagnetic power flow in the y-direction, the second the electromagnetic power flow in the z-direction, the third and fourth the kinetic power flows in the y-and z-directions, respectively. The right-hand side represents the kinetic powèr loss due to collisions.

§ 3. Dispersion Relation for a Thin Semiconductor Slab

We consider a semiconductor slab having the arrangement as shown in Fig. 3. We assume that the slab is thin enough. The electron beam in the semiconductor flows straight-forwards along it, because the Hall voltage is induced in the y-direction. Assuming that snaking arises slightly inside the both surfaces of the slab, we neglect the pressure gradient in the y-direction, although accumulation or absence of carriers comes out in the surface region.

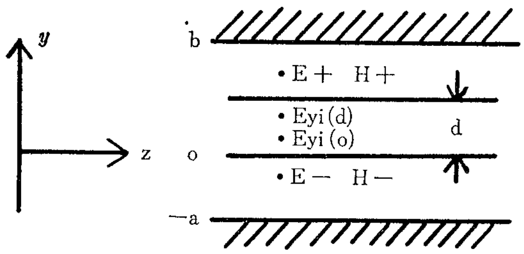

Fig. 3 The electron beam in the thin semiconductor slab. The suffixes +, − and i denote the corresponding quantities above, under and inside the electron beam, respectively. E. H. and d are the electric, magnetic field and thickness. The dashed boundaries are metals.

The equations of motion are given by[7)]

$$(j\omega + \nu)\,\bar{v}_{1y} + v_0 \frac{\partial}{\partial z}\,\bar{v}_{1y} = \eta \left\{ \bar{E}_{yi} + \frac{\rho_0\,\bar{y}_1}{\varepsilon_0} + \bar{v}_{1z}\, B_0 \right\}$$

$$(j\omega + \nu)\,\bar{v}_{1z} + v_0 \frac{\partial}{\partial z}\,\bar{v}_{1z} = \eta\,(\bar{E}_{1z} - \bar{v}_{1z}\, B_0) - \eta\,\frac{1}{\rho_0}\,\frac{\partial \bar{p}_1}{\partial z} \tag{3.1}$$

where

$$\eta = q/m^{*}$$

Since the slab is thin, quantities are represented by the average values denoted by bars above them. The electric field $\bar{E}_{yi}$ in eq. (3.1), which is the corresponding electric field inside the semiconductor, is written in the form[7)]

$$\bar{E}_{yi} = \frac{1}{2}\left\{ E_{yi}\ (d) + E_{yi}\ (o) \right\} = \frac{1}{2}\,(E_{y+} + E_{y-}) = \frac{\rho_0\,\bar{y}_1}{\varepsilon_0}$$

$$\equiv \bar{E}_{1y} - \frac{\rho_0\,\bar{y}_1}{\varepsilon_0} \tag{3.2}$$

where ε_0 is the permittivity of vaccum.
The continuity relation yields

$$\rho_1 = \frac{\beta \rho_0 \bar{v}_{1z}}{\omega - \beta v_0} \tag{3.3}$$

where β is the propagation constant.

The average electric field $\overline{E_{1y}}$ and $\overline{E_{1z}}$ are given by[7]

$$\overline{E_{1y}} = \frac{1}{2}(E_{y+} + E_{y-}) = \left\{ \frac{t}{\beta \varepsilon_0} \bar{\rho}_1 + \frac{\rho_0}{\varepsilon_0} s^2 \bar{y}_1 \right\}$$

$$\overline{E_{1z}} = j\left\{ \frac{r^2}{\varepsilon_0 \beta} \bar{\rho}_1 - \frac{\rho_0 t}{\varepsilon_0} \bar{y}_1 \right\} \tag{3.4}$$

where

$$r^2 = \frac{\sinh(\beta a) \sinh(\beta b)}{\sinh\beta(a+b)} \beta d$$

$$s^2 = \frac{\cosh(\beta a) \cosh(\beta b)}{\sinh\beta(a+b)} \beta d \tag{3.5}$$

$$t = \frac{\sinh\beta(a-b)}{2\sinh\beta(a+b)} \beta d$$

for the conditions $\beta a \gg 1$ and $\beta b \gg 1$, eqs. (3.5) are simplified as

$$r^2 = \beta d/2, \quad s^2 = \beta d/2, \quad t = 0 \tag{3.6}$$

The ac displacement y_1 is obtained by eqs. (3.1) and (3.3)

$$\bar{y}_1 = \frac{1}{j\Omega} \frac{j\eta \left[(\Omega - j\nu) - (\beta v_{th})^2/\Omega \right] \overline{E_{1y}} + \omega_c \eta \overline{E_{1z}}}{-(\Omega - j\nu)\left[(\Omega - j\nu) - (\beta v_{th})^2/\Omega\right] + \omega_c^2} \tag{3.7}$$

where

$$\Omega = \omega - \beta v_0, \ \eta = q/m^* \text{ and } \omega_c = \frac{qB_0}{m^*}$$

Using eqs. (3.1) through (3.7), we obtain a dispersion relation

$$\left[\left\{ \Omega(\Omega - j\nu) - (\beta v_{th})^2 - \omega_p{}^2 r^2 \right\} \left\{ \Omega(\Omega - j\nu) + \omega_p{}^2 s^2 \right\} - \left\{ \Omega\omega_c + \omega_p{}^2 t \right\}^2 \right]$$

$$\times \left[\left\{ (\Omega - j\nu) - \frac{(\beta v_{th})^2}{\Omega} \right\} (\Omega - j\nu) - \omega_c{}^2 \right] = 0 \tag{3.8}$$

where ω_p is the plasma frequency. The roots of eq. (3.8) are complicated. Accordingly the dioctron wave[7] which dominates the operation principle of the transverse field type electron beam is also much affected by thermal velocity and collision.

§ 4. Concluding Remarks

The complex Poynting theorem has been extended for transverse field type electron beam in solids of finite thickness to include collision and diffusion effect. An expected conclusion is obtained.

And dispersion relation is found for a thin semiconductor slab. For this case we assume dc accumulation or absence of carriers in the surface region, which induces the Hall voltages, does not strongly affect the carrier wave motion. However, a slight suppresion effect on the wave motion will actually occur in the surface region, especially in the accumulation side. Accordingly the ac pressure gradient term neglected in eq. (3.1) should be reexamined more in details when we study the effect of carrier density gradient in the y-direction, however, the condition assnmed here is considered to be reasonable enough as the first-order approximation.

The surface and bulk relaxation time are apparently different. In this paper, however, we introduced an equivalent collision frequency like the averaged one.

Temperature modulation effect and the velocity dependence of relaxation time should be considered, scince the electrons must be accelerated up to hot electron region enough to couple with the external circuit. Discussions on the temperature modulation effect will soon appear elsewhere.

Only the wave accompanied by the carrier is discussed in the present paper. However, in the course of the development of the solid state amplifier the external circuit becomes important and the very fine photoeching technique is also required.

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