# Introduction of Fractal Dimension Feature and Reduction of Calculation Amount in Person Authentication Using Evoked EEG by Ultrasound

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Abstract—The aim of this study is to authenticate individuals using an electroencephalogram (EEG) evoked by a stimulus. EEGs are highly confidential and enable continuous authentication during the use of or access to the given information or service. However, perceivable stimulation distracts the users from the activity they are carrying out while using the service. Therefore, ultrasound stimuli were chosen for EEG evocation. In our previous study, an Equal Error Rate (EER) of 0 % was achieved; however, there were some features which had not been evaluated. In this paper, we introduce a new type of feature, namely fractal dimension, as a nonlinear feature, and evaluate its verification performance on its own and in combination with other conventional features. As a result, an EER of 0 % was achieved when using five features and 14 electrodes, which accounted for 70 support vector machine (SVM) models. However, the construction of the 70 SVM models required extensive calculations. Thus, we reduced the number of SVM models to 24 while maintaining an EER = 0 %.

Index Terms—Biometric authentication, Electroencephalogram (EEG), Ultrasound, Fractal dimension, Calculation amount

### I. INTRODUCTION

In recent years, individuals can access various information from anywhere because of the spread of digital assistants such as mobile phones and tablet PCs. On the other hand, the outflow of personal information has increased, leading to extensive spoofing and other identity attacks. IC cards and passwords are commonly used to prevent those problems, but they can be stolen, lost or forgotten. Therefore biometric authentication, which is highly convenient for these purpose, is receiving extensive. Currently, biometric information such as fingerprints and iris images are used as authentication methods. However, these types of biometric information can be stolen because they are exposed on the body surface. In particular, as fingerprint marks remain where objects are touched due to the skin's sebum, the risk of theft is high. There is another problem when authenticating users using a password, IC card, fingerprint, or iris image: one-time-only authentication is assumed, which fails to prevent spoofing after the genuine user has been authenticated. To address this issue, it is necessary to authenticate a user continuously while using the system.

One of the biometric information that solves the problems mentioned above is the electroencephalogram (EEG). Since EEGs provide internal information about the body, they are highly confidential and there is low risk of leakage and theft. In addition, EEG is suitable for continuous authentication since it is constantly generated from the brain. EEGs are roughly divided into spontaneous EEG and evoked EEG. We focused on the evoked EEG generated by stimulation.

Regarding personal authentication using evoked EEG, there are some studies that use the subject's face picture as visual stimuli [1], and impose mental tasks such as calculation and imagination of figures [2]. However, these stimuli and tasks have the problem that they will interfere with the activity carried out by the users while using the system. In order to prevent this problem, the stimuli which evoke an EEG response must be imperceptible. Stimuli such as light, sound and vibration have a perceptible range, and stimuli outside of this range fail to be perceived. In the case of sound, human's audible range is 20 Hz-20 kHz [3]. In this study, in order to authenticate individuals using evoked EEG, we employed ultrasound, which is an auditory stimuli which cannot be perceived.

## II. PERSON AUTHENTICATION USING EVOKED EEG BY Ultrasound

The person authentication method using evoked EEG via ultrasound stimuli is explained below. In previous studies [4], [5], it was confirmed that ultrasound stimuli generated unique EEG responses. In addition, it was also confirmed that the authentication performance was improved especially in the  $\beta$  wave band when ultrasound stimulus created using the favorite songs of the experimental participants (personal ultrasound) were used to elicit more individual differences in the evoked responses. The personal ultrasound stimulus was created by removing the frequency components of 20 kHz or less of the participant's favorite song with a digital high pass filter. This ultrasound stimulus was played for 30 seconds in a silent room, according to the setup shown in Fig. 1. The participants were in a relaxed state with their eyes closed, and their EEG was measured. The EEG sensor used for the measurement was EPOC+ produced by EMOTIV in San Francisco, U.S.A. of whose sampling rate is 128 Hz and uses 14 electrodes positioned according to the extended international 10-20 system. The electrode positioning is shown on Fig. 2. The number of participants was 10 from male undergraduate and graduate students, and



Fig. 1. Measurement environment.



Fig. 2. Electrode position based on the extended international 10-20 system.



Fig. 3. An example of EEG data.

the EEG measurement was performed 8 times per participant. Fig. 3 shows an example of the measured raw EEG data, and such an EEG is obtained from each of the 14 electrodes.

The logarithmic spectrum (SP), which refers to the logarithm of the power spectrum of the EEG, was used as a feature to verify whether individuals were genuine or not. In order to analyze EEG which is a complex system, nonlinear features based on chaos: maximum lyapunov exponent (ML), sample entropy (SE), and permutation entropy (PE) were also used. Principal component analysis (PCA) was used for dimensional reduction and a support vector machine (SVM) was used for verification. The verification procedure for each feature and each electrode is illustrated in Fig. 4 and the total procedure for all features and electrodes is shown in Fig. 5.

The equal error rate (EER) was used for evaluation of the verification performance. The EER is the value when a false acceptance rate (FAR), that is, the rate of accepting



Fig. 4. Verification procedure for each feature and each electrode.

impostors, and a false rejection rate (FRR), that is, the rate of rejecting genuine users, are equal. The smaller EER, the better the authentication performance. As a result, EER = 0% was achieved by a majority decision on all features and electrodes. An SVM model was used for each feature and each electrode. Therefore, the total number of SVM models used to obtain this result was 56.

#### **III. INTRODUCTION OF FRACTAL DIMENSION FEATURE**

The brain is composed of a large number of neurons, and EEGs are considered as an output of such a complex system. In our previous study, nonlinear features (ML, SE, and PE) were used to analyze EEG [5]. However, there are some nonlinear features which have not been evaluated yet. In this paper, we introduce the fractal dimension (FD) feature based on chaos analysis and examine its effectiveness.

#### A. Fractal Dimension

Higuchi's method [6] is proposed for estimating fractal dimension of the time-series data. Since an EEG comprises time-series data, we apply Higuchi's method to estimate the fractal dimensions of the EEG, where the geometric complexity of time-series data is estimated. Higuchi's method is explained below.

First, the time-series of data N is expressed as

$$X(1), X(2), X(3), \dots, X(N).$$
 (1)

From these time-series data, datasets  $\widetilde{X}_m(k)$  defined by the following equation is reconstructed for k pieces.

$$\widetilde{X}_m(k); X(m), X(m+k), X(m+2k), \\ \dots, X\left(m + \left[\frac{N-m}{k}\right] \cdot k\right) \quad (m = 1, 2, \cdots, k), \quad (2)$$

where  $[\cdot]$  represents the Gauss symbol. Equation (2) means that time-series data with sampling time of  $\Delta t$  is sub-sampled



Fig. 5. Verification procedure for all features and electrodes.

with the sampling time  $k\Delta t$ . For example, in the case of k = 3 and N = 100,  $\tilde{X}_m(k)$  is re-expressed as follows:

$$X_1(3); X(1), X(4), X(7), \cdots, X(97), X(100), \widetilde{X}_2(3); X(2), X(5), X(8), \cdots, X(98), \widetilde{X}_3(3); X(3), X(6), X(9), \cdots, X(99).$$

Incidentally, the number of data may differ between datasets, e.g., such as 34 data in  $\widetilde{X}_1(3)$ , and 33 data in  $\widetilde{X}_2(3)$  and  $\widetilde{X}_3(3)$ .

Next, the length  $L_m(k)(m = 1, 2, ..., k)$  of each dataset  $\widetilde{X}_m(k)$  is calculated using the lengths of the two adjacent points of a dataset as

$$L_m(k) = \left\{ \left( \sum_{i=1}^{N-m} |X(m+ik) - X(m+(i-1)\cdot k)| \right) \frac{N-1}{\left[\frac{N-m}{k}\right]\cdot k} \right\} / k, (3)$$

where  $(N-1)/(\left[\frac{N-m}{k}\right] \cdot k)$  is a term that corrects the differences in the number of data in the reconstructed datasets, such as  $\widetilde{X}_1(3)$  and  $\widetilde{X}_2(3)$  in the above examples.

Next, the averaged length for all k pieces is given by  $\langle L(k) \rangle$  of the time-series data at k, which is expressed using the average of the k pieces of  $L_m(k)$  as

$$\langle L(k) \rangle = \frac{\sum_{m=1}^{k} L_m(k)}{k}.$$
(4)

Finally, the fractal dimension is obtained as an absolute value of the slope when k and  $\langle L(k) \rangle$  are plotted on a log-log graph.

## B. Preparation

The maximum value  $k_{\text{max}}$  of k and the length of data N must be determined when estimating the fractal dimension. Accardo et al. reported that, as a result of examining using the Weierstrass function and fractional Brownian motion, an optimal estimation value could be obtained by using  $k_{\text{max}} = 6$  and N > 125 in Higuchi's method [7]. These values are also used in this study. Furthermore, the straight line for finding the slope was fitted to the plot points using the least squares method.

## C. Evaluation

In order to verify the effectiveness of the fractal dimension features, comparisons with other nonlinear features were performed using the Euclidean distance matching method in two cases, in which each nonlinear feature was directly examined (one-dimensional case) and each nonlinear feature was multi-dimensionalized (multi-dimensional case). The cross-validation was performed 10 times in the verification.

The fractal dimension feature is a one-dimensional feature, as well as the other nonlinear features used in our previous study. Therefore, the number of dimensions is too small. In our previous study [5], the time-domain and frequency-domain methods were introduced to make a feature multi-dimensional. In the time-domain method, EEG data are divided equally into some segments, without overlapping, and feature extraction is performed at each segment. The number of segments varies between 2 and 10. In the frequency-domain method, using band pass filters, EEG data are divided into six frequency bands,  $\delta$  (0.4–4 Hz),  $\theta$  (4–8 Hz),  $\alpha$  (8–13 Hz), low  $\beta$  (13–20 Hz), high  $\beta$  (20–30 Hz), and  $\gamma$  (30–43 Hz), and a feature is extracted from each band. These methods were examined comprehensively

TABLE II
Eer [%] by One-dimensional nonlinear Features with Euclidean Distanci

	AF3	F7	F3	FC5	T7	P7	01	02	P8	T8	FC6	F4	F8	AF4	Mj
FD	37.4	40.9	39.1	38.8	43.6	33.6	35.7	33.3	31.6	33.7	33.2	33.8	36.4	31.4	28.7
ML	40.6	45.1	41.4	37.7	44.8	38.9	34.6	38.3	42.4	40.4	41.9	39.9	39.4	35.6	35.1
SE	37.5	39.8	38.6	44.5	40.8	37.2	38.3	38.9	32.9	35.3	34.8	38.9	38.8	37.6	31.3
PE	37.9	45.1	39.8	46.7	41.2	38.3	37.6	37.7	37.2	41.0	39.4	39.8	40.4	38.1	34.8

 TABLE III

 Eer [%] by Multi-dimensional Nonlinear Features with Euclidean Distance.

	AF3	F7	F3	FC5	T7	P7	01	O2	P8	T8	FC6	F4	F8	AF4	Mj
FD	37.9	39.6	39.6	37.7	43.0	33.8	34.9	31.7	29.6	35.6	32.4	34.6	35.6	32.1	27.6
ML	46.1	47.9	41.9	47.3	52.5	42.4	38.3	44.4	40.1	50.4	42.6	47.1	44.5	40.1	41.3
SE	36.9	38.9	39.3	45.9	40.5	38.5	41.4	36.7	30.3	34.9	34.6	37.7	38.4	36.1	31.2
PE	36.4	44.4	39.9	37.5	45.8	43.3	42.4	39.9	36.9	39.3	36.1	33.8	39.8	37.2	34.4

TABLE IV EER [%] BY MULTI-DIMENSIONAL NONLINEAR FEATURES WITH SVM.

AF3	F7	F3	FC5	T7	P7	01	O2	P8	T8	FC6	F4	F8	AF4	Mj
30.3	28.7	25.8	33.5	31.3	29.4	30.2	29.7	27.8	26.4	30.2	29.2	33.7	27.1	5.3
28.0	31.9	28.8	29.0	32.2	32.2	30.2	28.5	28.8	27.8	28.4	29.4	31.2	31.4	3.9
32.8	27.9	30.6	35.0	29.5	28.8	30.5	30.3	27.6	28.6	32.3	30.1	32.4	27.1	5.1
35.5	32.8	30.0	32.8	30.2	27.1	29.7	29.0	34.3	29.3	31.2	32.8	30.0	32.1	4.3
	AF3 30.3 28.0 32.8 35.5	AF3F730.328.728.031.932.827.935.532.8	AF3F7F330.328.725.828.031.928.832.827.930.635.532.830.0	AF3         F7         F3         FC5           30.3         28.7         25.8         33.5           28.0         31.9         28.8         29.0           32.8         27.9         30.6         35.0           35.5         32.8         30.0         32.8	AF3F7F3FC5T730.328.725.833.531.328.031.928.829.032.232.827.930.635.029.535.532.830.032.830.2	AF3         F7         F3         FC5         T7         P7           30.3         28.7         25.8         33.5         31.3         29.4           28.0         31.9         28.8         29.0         32.2         32.2           32.8         27.9         30.6         35.0         29.5         28.8           35.5         32.8         30.0         32.8         30.2         27.1	AF3         F7         F3         FC5         T7         P7         O1           30.3         28.7         25.8         33.5         31.3         29.4         30.2           28.0         31.9         28.8         29.0         32.2         32.2         30.2           32.8         27.9         30.6         35.0         29.5         28.8         30.5           35.5         32.8         30.0         32.8         30.2         27.1         29.7	AF3         F7         F3         FC5         T7         P7         O1         O2           30.3         28.7         25.8         33.5         31.3         29.4         30.2         29.7           28.0         31.9         28.8         29.0         32.2         32.2         30.2         28.5           32.8         27.9         30.6         35.0         29.5         28.8         30.5         30.3           35.5         32.8         30.0         32.8         30.2         27.1         29.7         29.0	AF3         F7         F3         FC5         T7         P7         O1         O2         P8           30.3         28.7         25.8         33.5         31.3         29.4         30.2         29.7         27.8           28.0         31.9         28.8         29.0         32.2         32.2         30.2         28.5         28.8           32.8         27.9         30.6         35.0         29.5         28.8         30.3         27.6           35.5         32.8         30.0         32.8         30.2         27.1         29.7         29.0         34.3	AF3         F7         F3         FC5         T7         P7         O1         O2         P8         T8           30.3         28.7         25.8         33.5         31.3         29.4         30.2         29.7         27.8         26.4           28.0         31.9         28.8         29.0         32.2         32.2         30.2         28.5         28.8         27.8           32.8         27.9         30.6         35.0         29.5         28.8         30.5         30.3         27.6         28.6           35.5         32.8         30.0         32.8         30.2         27.1         29.7         29.0         34.3         29.3	AF3         F7         F3         FC5         T7         P7         O1         O2         P8         T8         FC6           30.3         28.7         25.8         33.5         31.3         29.4         30.2         29.7         27.8         26.4         30.2           28.0         31.9         28.8         29.0         32.2         32.2         30.2         28.5         28.8         27.8         28.4           32.8         27.9         30.6         35.0         29.5         28.8         30.3         27.6         28.6         32.3           35.5         32.8         30.0         32.8         30.2         27.1         29.7         29.0         34.3         29.3         31.2	AF3         F7         F3         FC5         T7         P7         O1         O2         P8         T8         FC6         F4           30.3         28.7         25.8         33.5         31.3         29.4         30.2         29.7         27.8         26.4         30.2         29.2           28.0         31.9         28.8         29.0         32.2         32.2         30.2         28.5         28.8         27.8         28.4         29.4           32.8         27.9         30.6         35.0         29.5         28.8         30.5         30.3         27.6         28.6         32.3         30.1           35.5         32.8         30.0         32.8         30.2         27.1         29.7         29.0         34.3         29.3         31.2         32.8	AF3         F7         F3         FC5         T7         P7         O1         O2         P8         T8         FC6         F4         F8           30.3         28.7         25.8         33.5         31.3         29.4         30.2         29.7         27.8         26.4         30.2         29.2         33.7           28.0         31.9         28.8         29.0         32.2         32.2         30.2         28.5         28.8         27.8         28.4         29.4         31.2           32.8         27.9         30.6         35.0         29.5         28.8         30.5         30.3         27.6         28.6         32.3         30.1         32.4           35.5         32.8         30.0         32.8         30.2         27.1         29.7         29.0         34.3         29.3         31.2         32.8         30.0	AF3         F7         F3         FC5         T7         P7         O1         O2         P8         T8         FC6         F4         F8         AF4           30.3         28.7         25.8         33.5         31.3         29.4         30.2         29.7         27.8         26.4         30.2         29.2         33.7         27.1           28.0         31.9         28.8         29.0         32.2         30.2         28.5         28.8         27.8         28.4         29.4         31.2         31.4           32.8         27.9         30.6         35.0         29.5         28.8         30.3         27.6         28.6         32.3         30.1         32.4         27.1           35.5         32.8         30.0         32.8         30.2         27.1         29.7         29.0         34.3         29.3         31.2         32.8         30.0         32.1

 TABLE I

 Multi-dimensionalization Methods of Nonlinear Features.

Feature	Method (No. of Dimensions)
FD	Time-domain (4)
ML	Frequency-domain (6)
SE	Time-domain (8)
PE	Frequency-domain (6)

to determine the most suitable one. In this study, multidimensionalization of FD feature was also examined, and the multi-dimensionalization method with the lowest EER was chosen and compared with the other features.

## D. Result

Table I shows the multi-dimensionalization method evaluated and the number of dimensions for each feature. In the frequency-domain method, the number of dimensions is fixed at six.

Tables II and III show the EERs of one-dimensional and multi-dimensional nonlinear features when using Euclidean distance matching at each electrode and the EERs when the results of all electrodes for each feature were fused by a majority decision rule, respectively. AF3, F7, ..., AF4 are the electrode positions shown in Fig. 2, and Mj represents the case where the majority decision was applied to all electrodes. Although it is unclear whether FD is theoretically superior to other nonlinear features, the experimental results show that FD is superior to other nonlinear features when individually verified.

Next, the verification performance of FD was evaluated using an SVM. The multi-dimensionalization method was the same as the used in the verification with Euclidean distance matching. Table IV shows EERs for each electrode and by majority decision comparing with those by conventional nonlinear features. As a result, EERs by FD were smaller than those by conventional nonlinear features at some electrodes; however, an EER by Mj was the worst among four features while its degradation was slight.

In this study, FD was introduced as a new nonlinear feature in the proposed method; however, its verification performance was equivalent to those of the other nonlinear features. Thus, the results by the FD feature were fused with those by conventional features, SP, SE, ML, and PE based on the majority decision rule. Because 14 electrodes and five features were used in this verification, the number of SVM models used was 70. FAR and FRR curves were plotted in Fig. 6. As a result, EER = 0% was achieved when the thresholds were set to 36, 37, and 38.

### IV. REDUCTION OF CALCULATION AMOUNT

EER = 0% was achieved by using five features; however, it required 70 SVM models<sup>1</sup>. Fewer SVM models was necessary for reducing the calculation amount of the proposed method.

Therefore, we investigated how much the number of SVM models, that is, the number of features and electrodes, could be reduced while maintaining EER = 0%. The most reliable method for feature and electrode selection is brute force evaluation. The number of combinations of SVM models is calculated as

$$\sum_{r=1}^{N} {}_{N}C_{r} = 2^{N} - 1, \tag{5}$$

where N is the number of SVM models used. It resulted in approximately  $1.2 \times 10^{21}$  for N = 70. The time required to perform brute force evaluation using only one feature, that is, around 16,000 combinations, was approximately 0.8 hour<sup>2</sup>; therefore, it is impossible to perform brute force evaluation for all combinations. Consequently, features and electrodes used for verification were selected by the following condition.

<sup>&</sup>lt;sup>1</sup>56 SVM models were required in a previous study [5].

 $<sup>^{2}</sup>$ The PC used for this verification consisted of an intel<sup>®</sup> Xeon<sup>®</sup> processor E3-1241 v3 (8M cache, 3.50 GHz) and 16 GB of RAM.



Fig. 6. Error rate curves when fusing five features.

 TABLE V

 Combination of Electrodes and Features Used.

Feature		Electrode														
Teature	1	2	3	4	5	6	7	8	9	10	11	12	13	14		
1		0	0			0		0		0	0	0		0		
2		0	0			0		0		0	0	0		0		
3																
4		0	0			0		0		0	0	0		0		
5																

TABLE VI Combinations of Minimal Number of Electrodes and Features Keeping EER = 0%.

Features	Electrodes
SP, ML, SE, PE	AF3, AF4, F3, FC6, P7, P8
SP, ML, SE, PE	AF3, AF4, F7, FC6, P7, P8
SP, ML, PE, FD	AF3, F7, P7, P8, T8, AF4
SP, ML, PE, FD	AF3, F7, F3, P7, T8, FC6
SP, ML, SE	AF3, F3, F7, FC6, T8, P7, P8, O2
SP, ML, SE	AF3, F3, F7, FC6, T7, T8, P8, O2
SP, ML, FD	AF3, F7, T7, P7, O1, P8, T8, F4
SP, ML, FD	F7, F3, T7, P7, O1, P8, T8, F4

## A. Condition for Selection

Even if the number of electrodes is reduced, if the combination of electrodes is different for each feature, the user will eventually have to wear all the electrodes. When the combination of electrodes used is identical in all features, there is no need to wear all electrodes, which is more convenient for the users. Therefore, the electrodes used were selected under the condition that the combination of electrodes was identical for all the features used.

Examples of combining features and electrodes selected by this method is shown in Table V, where "O" indicate the selected electrodes.

The total number of combinations investigated by this selection method is given by the product of the number of combinations that selects m from five features and the number of combinations that selects n from 14 electrodes.

$$\sum_{m=1}^{5} {}_{5}C_{m} \times \sum_{n=1}^{14} {}_{14}C_{n} = 31 \times 16,383 = 507,873.$$
(6)

This number of evaluations can be achieved in a realistic time.

## B. Result

Table VI shows the minimal number of combinations of features and electrodes that enable an EER of 0%. When six electrodes were used for four features or eight electrodes were used for three features, EER = 0% could be maintained. The number of features and electrodes used corresponds to that of the SVM models used for verification. As a result, it was confirmed that 70 SVM models can be reduced to 24 keeping EER = 0%.

#### V. CONCLUSIONS

The purpose of this study was to authenticate individuals using evoked EEG by ultrasound. In this paper, we introduced a fractal dimension feature and evaluated its verification performance. As a result, the verification performance of FD was superior to those of nonlinear features used in our previous study when Euclidean distance matching was used. On the contrary, the verification performance of FD feature was slightly degraded when using SVM. Fusing the results of five features, SP, ML, SE, PE, and FD, achieved an EER of 0 % using 70 SVM models. However, the construction of all SVM models required extensive calculation. The number of SVM models corresponds to that of features and electrodes used for verification. Thus, the reduction of features and electrodes used for verification was examined. As a result, the number of SVM models could be reduced to 24 keeping an EER of 0% under the condition that the same combination of electrodes was used for each feature.

All of the features currently in use are obtained from a single electrode. However, as mentioned in Section III, EEG is a complex system composed of a large number of neuron. In the future, it is necessary to introduce mutual features that are obtained by analyzing multiple electrodes in order to extract more characteristic feature of EEG.

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