NONLINEAR RESPONSE ANALYSES OF RECTANGULAR RIGID BODIES SUBJECTED TO HORIZONTAL AND VERTICAL GROUND MOTION

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SUMMARY

This investigation deals with nonlinear seismic responses of free-standing rectangular rigid bodies on the horizontally and vertically accelerating rigid foundation. The responses are classified into two initial responses and four subsequent responses, accordingly the equations of motion governing the liftoff, slip and liftoff-slip interaction motions and boundary conditions specifying commencement and termination of the motions are defined.

The time histories of responses presented herein show that the body is sensitive to small changes in the friction coefficient and slenderness and to the wave properties and intensity of ground motions. Systematic trends are observed: The bodies on the low grip foundation avoid overturning while they are allowed to slip regardless of details of ground motions; The long period earthquakes tend to make the body overturn and slip largely. In contrast, the timing when liftoff and slip commences and

terminates and their directions do not directly correspond with intensity of ground motions. Moreover, the vertical ground motion adds irregularities on the responses, since it encourage or discourage to begin and end the responses.

It is concluded that governing equations of motion and boundary conditions in view of discontinuous nonlinear systems are necessary to analyze actual motions of the rectangular rigid bodies subjected to horizontal and vertical ground motion.

KEYWORDS

free-standing rectangular rigid body, liftoff motion, slip motion, liftoff-slip interaction motion, overturning, discontinuous nonlinear response system

INTRODUCTION

The systematically dynamics of a rigid block on a rigid foundation undergoing horizontal motion was investigated by Houser (1963). The minimum accelerations of a rectangular pulse and a half-cycle since-wave required to overturn the block without the associated impacts were presented. Yim et al. (1980) derived the governing equations of motion of the rocking block subjected to horizontal and vertical ground motion and computed its response based on the linearized equations of motion. The sensitivity of response to system and ground motion properties was studied. Spanos et al. (1984) carried out nonlinear analysis of rocking motion and examined nonlinear effects on judgement of overturning. Unfortunately, the slip motion was neglected in these investigations. Ishiyama (1982) and Shenton et al. (1991) classified the response of the body subjected to horizontal and vertical base excitations into five modes (rest, slide, rock, slide-rock and free flight) and their governing equations of motion and criteria were presented. Shenton (1996) derived criteria for initiation of a slide, a rock and a slide-rock mode for a block and pointed out the finite angular acceleration at the instant when the block begins to rock and slide-rock modes. Pompei et al. (1998) and Zhang et al. (2001) presented the condition needed to avoid sliding during the entire rocking motion by incorporating rocking exerted horizontal and vertical reaction forces into the criteria. Unfortunately, these investigations neglected the effects of vertical ground motion, which might be extremely important to discuss responses and criteria.

On the contrary, the writer(s) independently derived the governing equations of motion of slip, liftoff and liftoff-slip interaction and also equations of associate base shear and vertical reaction force including liftoff motion exerted forces. In addition, the analytical results including post-impact motion under an assumption of a small liftoff angle are compared with measurements during the free and horizontally accelerated liftoff motion tests [9] and during the free liftoff-slip interaction motion test [10].

The following paper presents a consistent formulation governing the responses of free-standing rigid bodies to horizontal and vertical base excitations based on previous results presented by writer(s). The formulation is based on the assumption of a rectangular rigid body, rigid foundation, Coulomb friction, impact on classical analytical dynamics and no flight mode.

Firstly, the governing equations of motion and boundary conditions for a slip, a liftoff and a liftoff-slip interaction motion are derived by considering horizontal and vertical ground motion effects.

Secondly, the derived governing equations of motion and the boundary conditions are compared with the results by earlier investigators to contrast effects of vertical ground motion on the response and sufficient friction needed to avoid slipping during a liftoff motion.

Finally, by using two recorded accelerograms, some nonlinear analyses are carried out to examine effects of earthquake properties on the response. Their intensities and the value of friction coefficients are also varied. In addition, to highlight effects of the vertical ground motion on the responses, the intensity of vertical ground motion is intentionally varied.

CLASSIFICATION OF SEISMIC RESPONSES

Based on careful observations, seismic responses of a rectangular rigid body on the rigid foundation are classified according to the response history and its behavior. Figure 1 shows the schematic classification of response states. Initially, the body has the possibility of behaving either the liftoff or the slip motions, when subjected to base excitations. The subsequent response is either the liftoff, the slip or the liftoff-slip interaction motions. Since further responses come under either of them, this classification covers all response states that inevitably occur. These response states and transition conditions are clearly identified by governing equations of motion and boundary conditions that specify when the motion of interest commences or terminates. Notes under each illustration demonstrate the kinds of equations of motion and boundary conditions that are necessary to describe the state classified. It is worth to point out that transition conditions are different according to the previous state, although the response arrives at the same state. The same suffix on the right shoulder indicates whose equations of motion or boundary conditions are identical.

LIFTOFF MOTION

Equations of motion

A body subjected to horizontal and vertical ground accelerations of the rigid base is shown in rotated position in Figure 2(a). In this state, the coefficient of friction is assumed to be sufficiently large so that there will be no slip between the body and the foundation. The body will rotate around the left and right bottom edge by turns. The equations of motion of the body, governing the liftoff angle $\ddot{\theta}_r$ and $\ddot{\theta}_l$ from the horizontal, subjected to horizontal and vertical ground accelerations $\ddot{z}_h$ and $\ddot{z}_v$ are derived by considering the equilibrium of moments about the centers of rotation, respectively.

(a) While pivoting left bottom edge

$$I_0\ddot{\theta}_r = -M(g + \ddot{z}_v)R\sin(\alpha - \theta_r) + M\ddot{z}_h R\cos(\alpha - \theta_r) \tag{1}$$

(b) While pivoting right bottom edge

$$I_0\ddot{\theta}_l = -M(g + \ddot{z}_v)R\sin(\alpha - \theta_l) - M\ddot{z}_h R\cos(\alpha - \theta_l) \tag{2}$$

(c) Unification of equations of motion

Since these two equations have the same structure, by introducing the index λ, which specifies the rotational direction, they can be unified as follows.

$$I_0\ddot{\theta} = -M\left(g + \ddot{z}_v\right)R\sin(\alpha - \theta) + \lambda M\ddot{z}_h R\cos(\alpha - \theta) \tag{3}$$

Here, M, g and I_0 are mass of the body, gravity acceleration and moment of inertia of the body around the either bottom edges, for rectangular blocks, $I_o = 4MR^2/3$, respectively. The index shall possess a function as both identification of pivoting edge and determination of seismic inertia force actions. Accordingly, the value of λ is 1 when the body pivots on the left bottom edge, and is –1 when it does on the right bottom edges. λ changes its sign every landing of either bottom edges. Employing λ and θ, the liftoff angle θ_r and θ_l can be expressed as follows.

$$\theta_r = \theta, \quad \theta_l = 0 \quad (\text{at } \lambda = 1)$$

$$\theta_r = 0, \quad \theta_l = \theta \quad (\text{at } \lambda = -1) \tag{4}$$

(d) While stationary

The equation of motion for stationary condition is;

$$\ddot{\theta} = 0 \tag{5}$$

Liftoff commencement conditions

The liftoff commencement conditions of the rectangular rigid body subjected to horizontal and vertical ground acceleration are derived from the equilibrium of the overturning moment and the resistance moment around a pivoting edge.

$$M\left(g+\ddot{z}_v\right)R\sin\alpha < M\left|\ddot{z}_h\right| R\cos\alpha \tag{6}$$

The initial value of λ is given as follows based on the judgement, which edge liftoffs.

$$\ddot{z}_h > 0 \quad \text{and} \quad \left(g+\ddot{z}_v\right)\tan\alpha < \left|\ddot{z}_h\right| \quad \text{then;} \quad \lambda = 1 \tag{7.a}$$

$$\ddot{z}_h < 0 \quad \text{and} \quad \left(g+\ddot{z}_v\right)\tan\alpha < \left|\ddot{z}_h\right| \quad \text{then;} \quad \lambda = -1 \tag{7.b}$$

Restitution conditions

The transition from the liftoff around an edge to the liftoff around another one is accompanied by an impact. The associated energy loss is accounted for by reducing the angular velocity of the body after impact. Specifically, it can be expressed as follows in compliance with law of conservation of momentum.

$$\dot{\theta}\left(t^+\right) = e\dot{\theta}\left(t^-\right) \qquad 0 \le e \le 1 \tag{8}$$

Whereas e is coefficient of restitution; t^+ is the time just after impact; t^- is the time just before impact. Although the duration of impact was measured at free liftoff motion tests [10], it is ignored in this study. Changes in angular velocity are considered to occur instantaneously.

Liftoff termination conditions

Since there are no conditions that specify the termination of the liftoff motion except the liftoff angle computed, the liftoff motion continues until the kinetic energy in the response system completely dissipates. The response system looses its kinetic energy every landing of either bottom edges as restitution law specifies.

Overturning conditions

The overturning is judged by the liftoff angle computed. A range of liftoff angle θ is $0 \le \theta \le \pi/2$. Therefore, the body in standing position is $\theta = 0$ while it in the overturning position is $\theta = \pi/2$. Moreover, λ helps to identify the direction, which the body is overturned.

SLIP MOTION

Equations of motion

A body subjected to horizontal and vertical ground accelerations of the rigid base is shown in translated position in Figure 2(b). In this state, the foundation allows the body to slip without any hesitation except the friction force between the body and the foundation. The equations of motion of the body, governing the slip displacement x from the vertical, subjected to horizontal and vertical ground accelerations $\ddot{z}_h$ and $\ddot{z}_v$ are derived by considering the equilibrium of forces.

(a) While slipping

$$M\ddot{x} = -M\ddot{z}_h - F \tag{9.a}$$

$$F = \nu\, sign(\dot{x}) M(g + \ddot{z}_v) \tag{9.b}$$

in which ν is the kinetic friction coefficient. F is the kinetic friction force on which acts the body. $sign(\dot{x})$ is function which gives the sign of variables.

(b) While stationary

The equation of motion while stationary condition is;

$$\ddot{x} = 0 \tag{10}$$

Slip commencement conditions

The slip commencement conditions of the body subjected to horizontal and vertical accelerations are determined by the equilibrium of the horizontal seismic inertia force and the static friction force.

$$\mu M\left(g + \ddot{z}_v\right) < M\left|\ddot{z}_h\right| \tag{11}$$

Whereas μ is the coefficient of static friction.

Slip termination condition

The slip continues until the relative velocity between the body and the foundation becomes zero. The slip termination conditions are, therefore, given as follows.

$$\dot{x} = 0 \tag{12}$$

LIFTOFF-SLIP INTERACTION MOTION

Equations of motion

A body subjected to horizontal and vertical ground accelerations of the rigid base is shown in rotated and translated position in Figures 3 and 4. Moreover, the liftoff motion and ground motion induced forces are also demonstrated. In this state, the foundation allows the body to liftoff and slip without any hesitation except the friction force between the body and the foundation. In accordance with variational approach proposed by writer(s) [10], the equations of motion of the body, governing the liftoff angle $\ddot{\theta}$ from the horizontal with the index λ and the slip displacement x from the vertical, subjected to horizontal and vertical ground accelerations $\ddot{z}_h$ and $\ddot{z}_v$ are derived. It is assumed that the friction force F acts to prevent the virtual horizontal slip displacement from extending.

Lagragian of the system L is calculated as;

$$L = \frac{1}{2}M\dot{x}^2 - \lambda MR\cos(\alpha-\theta)\dot{x}\,\dot{\theta} + \frac{1}{2}\left(MR^2 + I\right)\dot{\theta}^2 - Mg\{R\cos(\alpha-\theta) - R\cos\alpha\} \tag{13}$$

To determine generalized forces on the system, the virtual work is examined. The horizontal seismic inertia force $-M\ddot{z}_h$ acts on the gravity center of the body and affects both the virtual slip displacement and the virtual horizontal displacement exerted by the liftoff motion, that are given as $\delta x - \lambda R\cos(\alpha-\theta)\,\delta\theta$. On the contrary, the kinetic friction force F acts on the pivoting edge to prevent the body from slipping. Then, it affects only the virtual slip displacement δx. Moreover, the vertical seismic inertia force $-M\ddot{z}_v$ acts on the gravity center of the body and affects the virtual vertical displacement exerted by the liftoff motion, that is given as $R\sin(\alpha-\theta)\,\delta\theta$. Therefore, the virtual work $\overline{\delta W}$ is derived as;

$$\overline{\delta W} = \left(-M\ddot{z}_h - F\right)\delta x + \{\lambda M\ddot{z}_h R\cos(\alpha-\theta) - M\ddot{z}_v R\sin(\alpha-\theta)\}\delta\theta \tag{14}$$

Therefore, Lagrange's equations of motion are derived as follows.

rotational direction; $$I_0\,\ddot{\theta} - \lambda MR\cos(\alpha-\theta)\,\ddot{x} = -M\left(g + \ddot{z}_v\right)R\sin(\alpha-\theta) + \lambda M\ddot{z}_h R\cos(\alpha-\theta) \tag{15}$$

horizontal direction; $$M\ddot{x} - \lambda MR\left\{\sin(\alpha-\theta)\,\dot{\theta}^2 + \cos(\alpha-\theta)\,\ddot{\theta}\right\} = -M\ddot{z}_h - F \tag{16}$$

Here, $MR^2 + I = I_0$, I : moment of inertia of the body around the gravity center. The definition of the kinetic friction force F is discussed by considering the effects of liftoff motion exerted forces at the later section.

Slip commencement conditions including liftoff motion effects

The rotational forces associated with the liftoff motion induce the base shear and vertical reaction force. By taking the effects of rotational forces and the action of seismic inertia forces into account, the slip commencement conditions are defined. Consider the inclined $x' - y'$ plane by the liftoff angle of the body θ_r and θ_l from horizontal shown in Figures 3 and 4. Introduce a pair of forces $R_{x'}$ and $R_{y'}$ on an x'-axis and a y'-axis to prevent pivoting edges of the body from slipping. By employing the index λ, these are;

$$R_{x'} = \lambda Mg\sin\theta - \lambda MR\dot{\theta}^2\sin\alpha - \lambda MR\ddot{\theta}\cos\alpha + M\ddot{z}_h\cos\theta + \lambda M\ddot{z}_v\sin\theta \tag{17}$$

$$R_{y'} = Mg\cos\theta - MR\dot{\theta}^2\cos\alpha + MR\ddot{\theta}\sin\alpha - \lambda M\ddot{z}_h\sin\theta + M\ddot{z}_v\cos\theta \tag{18}$$

Then $R_{x'}$ and $R_{y'}$ are transformed into the forces R_x and R_y on the global coordinate to judge the slip commencement. A pair of forces needed to avoid slipping on the global coordinate is derived as follows.

$$R_x = -\lambda MR\dot{\theta}^2\sin(\alpha-\theta) - \lambda MR\ddot{\theta}\cos(\alpha-\theta) + M\ddot{z}_h \tag{19}$$

$$R_y = Mg - MR\dot{\theta}^2\cos(\alpha-\theta) + MR\ddot{\theta}\sin(\alpha-\theta) + M\ddot{z}_v \tag{20}$$

The similar expressions of Eqs. (19) and (20) can also be found in the papers by Pompei et al. (1998) and Zhang et al. (2001) in case of $\ddot{z}_v$ =0 and λ =1. Since the static friction force is given as the product of the static friction coefficient and the vertical reaction force, the slip commencement conditions of the body including liftoff motion effects are derived as follows.

$$\mu\left\{M\left(g+\ddot{z}_v\right)-MR\dot{\theta}^2\cos(\alpha-\theta)+MR\ddot{\theta}\sin(\alpha-\theta)\right\}<\left|M\ddot{z}_h-\lambda MR\dot{\theta}^2\sin(\alpha-\theta)-\lambda MR\ddot{\theta}\cos(\alpha-\theta)\right| \quad (21)$$

Whereas μ is the static friction coefficient.

Kinetic friction force during slip including liftoff motion effects

From an analogy with the vertical reaction force presented at previous section, the kinetic force during the slip of the body including liftoff motion effects is given as follows.

$$F=\nu\, sign(\dot{x})\left\{M\left(g+\ddot{z}_v\right)-MR\dot{\theta}^2\cos(\alpha-\theta)+MR\ddot{\theta}\sin(\alpha-\theta)\right\} \quad (22)$$

Whereas ν is coefficient of kinetic friction.

Liftoff commencement conditions during slip

An analogy with the liftoff commencement conditions for the initial response, the liftoff commencement conditions during slip can be derived by incorporating inertia forces due to the slip motion into the liftoff commencement condition defined by Eq. (6). From the equilibrium of the overturning moment and the resistance moment, the liftoff commencement conditions during slip are given as;

$$M\left(g+\ddot{z}_v\right)R\sin\alpha < M\left|\ddot{x}+\ddot{z}_h\right|R\cos\alpha \tag{23}$$

The initial value of the index λ is given as follows.

$$\ddot{z}_h+\ddot{x}>0 \quad \text{and} \quad \left(g+\ddot{z}_v\right)\tan\alpha < \left|\ddot{x}+\ddot{z}_h\right| \quad \text{then;} \quad \lambda = 1 \tag{24.a}$$

$$\ddot{z}_h+\ddot{x}<0 \quad \text{and} \quad \left(g+\ddot{z}_v\right)\tan\alpha < \left|\ddot{x}+\ddot{z}_h\right| \quad \text{than;} \quad \lambda = -1 \tag{24.b}$$

COMMENTS ON VERTICAL GROUND MOTION EFFECTS

This section highlights effects of vertical ground motion on governing equations of motion and boundary conditions by comparing with results by earlier investigators. The following results can be found in the paper by Shenton (1996) and Zhang et al. (2001) in case of A_v=0 and λ=1 and demonstrate generalities of proposed methods.

At initiation of motions

At the instant when the block initiates liftoff motion ($\theta \approx 0$), Eq. (3) can be rewritten as follows by employing notation introduced by Shenton (1996).

$$\ddot{\theta} = \frac{3g\left\{\lambda\gamma A_h - \left(1+A_v\right)\right\}}{4B\left(\gamma^2+1\right)} \tag{25}$$

Here, $B = R\sin\alpha$, $H = R\cos\alpha$, $r = H/B$, $A_h = \ddot{z}_h/g$, $A_v = \ddot{z}_v/g$. The substitution of Eq. (25) into Eq. (21) and an assumption at initiation of liftoff motion ($\theta = \dot{\theta} \approx 0$) gives sufficient friction to prevent slipping upon the initiation of a liftoff motion.

$$\left|\frac{R_x}{R_y}\right| = \left|\frac{3\lambda\gamma(1+A_v)+A_h\gamma^2+4A_h}{4\gamma^2+1+3\lambda A_h\gamma+A_v(4\gamma^2+1)}\right| > \mu \tag{26}$$

From these equations, the vertical ground acceleration can not be ignored, since it contributes to the angular acceleration and sufficient friction at initiation of a liftoff motion.

In addition, Solving Eqs. (15), (16) and (22) for $\ddot{\theta}$ and $\ddot{x}$;

$$\ddot{\theta} = \frac{-3g(1+A_v)(1+\lambda\nu\, sign(\dot{x})\gamma)}{B(\gamma^2+4+3\lambda\nu\, sign(\dot{x})\gamma)} \tag{27}$$

$$\ddot{x} = -g\frac{(1+A_v)(3\lambda\gamma+\nu\, sign(\dot{x})+4\nu\, sign(\dot{x})\gamma^2)+A_h(\gamma^2+3\lambda\nu\, sign(\dot{x})\gamma+4)}{\gamma^2+4+3\lambda\nu\, sign(\dot{x})\gamma} \tag{28}$$

The vertical ground acceleration also significantly contributes to the angular acceleration at initiation of a liftoff-slip interaction motion, although effects of horizontal ground acceleration on the angular acceleration are eliminated.

On the contrary, changes of criteria for slip and liftoff and liftoff-slip modes due to vertical ground acceleration are illustrated in Figure 6. The original criteria were developed by Shenton (1996) without considering the action of vertical ground acceleration. Vertical ground acceleration changes horizontal ground acceleration required to enter liftoff motion. Despite the value of the vertical acceleration, a sufficient friction required to avoid slipping at smallest horizontal acceleration is constant. In addition, the direction of vertical ground acceleration affects distribution of sufficient friction.

Under liftoff motions

By employing notations proposed by Zhang et al. (2001), Eq. (3) can also be expressed in the compact form.

$$\ddot{\theta} = -p^2 \left\{ \left(1 + \frac{\ddot{z}_v}{g} \right) \sin(\alpha - \theta) - \lambda \frac{\ddot{z}_h}{g} \cos(\alpha - \theta) \right\} \tag{29}$$

Here, $p^2 = 3g/4R$. In addition, the substitution Eq. (29) into Eq. (21) gives compact form of a sufficient friction to avoid slipping under liftoff motion.

$$\left| \frac{R_x}{R_y} \right| = \left| \frac{\ddot{z}_h \{5 - 3\cos 2(\alpha - \theta)\} + 3\lambda (g + \ddot{z}_v) \sin 2(\alpha - \theta) - \lambda \frac{6g}{p^2} \dot{\theta}^2 \sin(\alpha - \theta)}{3\lambda \ddot{z}_h \sin 2(\alpha - \theta) + (g + \ddot{z}_v)\{5 + 3\cos 2(\alpha - \theta)\} - \frac{6g}{p^2} \dot{\theta}^2 \cos(\alpha - \theta)} \right| > \mu \tag{30}$$

From Eq. (29), the vertical ground acceleration directly works restoring force in the system during the entire liftoff motion. From Eqs. (26) and (30), since vertical ground accelerations affect both base shear and vertical reaction force, it has to be considered at discussing actual behavior of the body.

RESULTS AND DISCCUSIONS

Some numerical calculations are presented using a rectangular rigid body whose height, width and mass are 0.2m, 0.1m and 10kg, respectively. The body is set on the flat and smooth rigid foundation whose static fiction coefficient is adjusted for the purpose of the analysis. The kinetic friction coefficient is fixed as 95 % of the static friction coefficient specified in each analytical case. The coefficient of restitution is 0.8. Following two recorded accelerograms are used to contrast effects of seismic wave properties on the responses of the body.

Mexico, Mexico; 19 September 1985.

(maximum horizontal acceleration; 94.6gal, maximum vertical acceleration; 27.7gal)

Hyogoken-nanbu, Kobe; 17 January 1995.

(maximum horizontal acceleration; 818.0gal, maximum vertical acceleration; 332.2gal)

Mexico earthquake is classified as long period earthquake while Hyogoken-nanbu earthquake is classified as short period earthquake. The earthquake intensity is adjusted to the specified horizontal acceleration level with maintaining relationships between recorded maximum horizontal and vertical accelerations except final analysis series.

Table 1 lists the analysis conditions, judgement of overturning, brief description about significant events observed according to figure number. If the same responses are observed despite different analysis conditions, either one of results is demonstrated as their typical responses. All figures have magnified time window where events are observed and have two vertical axes that show the liftoff angle and the slip displacement by the left and the right axes, respectively.

Using simulation language ACSL [11], the nonlinear numerical calculations are carried out with 0.0001 seconds calculation intervals to accurately judge hitting the foundation. Since accelerograms are collected in 0.02 seconds interval, the seismic acceleration data between data acquisition intervals are linearly interpolated.

Figures 6(a), 7(a), 8(a) and 9(a) imply that the body on the low grip foundation tends to slip during an earthquake. From the comparison of Figures 6(a) with 7(a) and of Figures 8(a) with 9(a), stronger accelerations tend to produce large slip displacement. However, their tracks, when the slip commences and terminates, which direction the body slips, are much different. It seems difficult to find regularities in them even though subjected to the same earthquake. In addition, from the comparison of Figures 6(a) with 8(a) and of Figures 7(a) with 9(a), the long period earthquake tends to induce large slip displacement because favorable accelerations for a slip last for a while. Moreover, the body on the low grip foundation may avoid overturning (Figure 9(a)) while the body on high grip foundation is overturned by the same intensity earthquake (Figure 9(c)). It is reasoned from Eqs. (26) or (30) that the body can not enter or maintain a liftoff motion on the insufficient friction foundation.

Generally, it is considered that the risk of overturning will increase as horizontal acceleration increases. Within results presented herein, the body avoids overturning when maximum horizontal acceleration is 600 gal (Figure 8(c)) while the body is overturned when maximum horizontal acceleration is 800 gal (Figure 9(c)). However, in Figures 6(b) and 7(b), the body is overturned regardless of acceleration intensity when Mexico earthquake is applied, although the timing of overturning is much different by the acceleration intensity. In addition, the overturning can also be found at low static friction coefficient (Figures 6(b) and 7(b)), which the body avoids overturning when Hyogoken-nanbu earthquake is applied (Figures 8(b) and 9(b)). Its primary reason is that the favorable accelerations for a liftoff last enough to overturn the body. In contrast, form the comparison of Figures 6(b) with 8(b) and of Figures 7(b) with 9(b), vertical ground accelerations may accidentally help to avoid overturning by introducing a liftoff-slip interaction motion in case of Hyogoken-nanbu earthquake. The vertical accelerations may reduce sufficient friction. On the contrary, the overturning occurs without experience of growing liftoff response (Figures 6(b), 7(b) and 10(c)). This is a particular point, which is completely different from resonance in vibration problem, and makes governing equations a view from discontinuous system need.

The liftoff-slip interaction motions are observed not only about 33.5 and 35.5 seconds in Figure 8(b) and also about 32.5, 33.2, 35.5 and 35.9 seconds in Figure 9(b). The difference of their appearance timing depends on the intensity of accelerations used. To highlight the liftoff-slip interaction effects on the slip displacement, an additional analysis is carried out. Figure 10(a) shows results under the same analysis conditions of Figure 9(b) except eliminating the liftoff motion of the body. To eliminate the liftoff motion, the body in overturned sate is used. From the comparison of them about 32.5, 33.2, 35.5 and 35.9 seconds, it can be found that the liftoff-slip interaction motion reduces the slip displacement of the body. This can be deduced from the view of the mechanical energy balance inherent in the system [10].

The action of vertical accelerations is considered to have great influences on a liftoff, a slip and a liftoff-slip interaction motion of the body as many previous investigators pointed out [for example; 2 and 5]. Figures 10(b) and 10(c) are results under the same analysis conditions of Figure 9(b) except magnifying scale of vertical accelerations. Figure 10(b) shows results under half intensity of vertical accelerations while Figure 10(c) demonstrates results under twice intensity of vertical accelerations. The liftoff angle about 32.5 seconds becomes a tenth liftoff angle in Figure 10(c) but the same one in Figure 10(b). However, in Figure 10(b), the large liftoff angle suddenly appears about 33.5 seconds and its rather irregular rebounds continue to 34.7 seconds. The varying vertical accelerations contribute to rebound irregularities as Eq. (29) implies. In addition, slip tracks become much different. The slip displacement about 33.5 seconds is reduced by liftoff-slip interaction in Figure 10(b) while it is extended by the action of vertical ground acceleration in Figure 10(c).

Since responses of the rectangular rigid body are completely discontinuous and irregular, governing equations of motion inspiring from nonlinear discontinuous systems are necessary to calculate the actual seismic responses of the body.

CONCLUSIONS

The principle conclusions of this study concerned with dynamics of rigid body on the rigid foundation may be summarized as follows.

The body on the low grip foundation may avoids overturning even though subjected to high intensity base excitations, while it should be allowed to have large slip displacement. In contrast, the body on the high grip foundation may be overturned. The sufficient friction can be used to judge the grip condition during entire motions.

The long period earthquake may rise a risk of large slip displacement and overturning of the body, since it may possess prolonged favorable horizontal accelerations for a slip and a liftoff.

The liftoff-slip interaction motion may occur in limited conditions and may reduce the slip displacement. The vertical accelerations accidentally help to begin the liftoff-slip interaction motion by reducing sufficient friction.

Since vertical accelerations add irregularities to responses of the body, it can not be ignored when evaluating the responses of the body. In addition, to simulate actual motions of the body, governing equations of motion derived from a view of nonlinear discontinuous systems is necessary.

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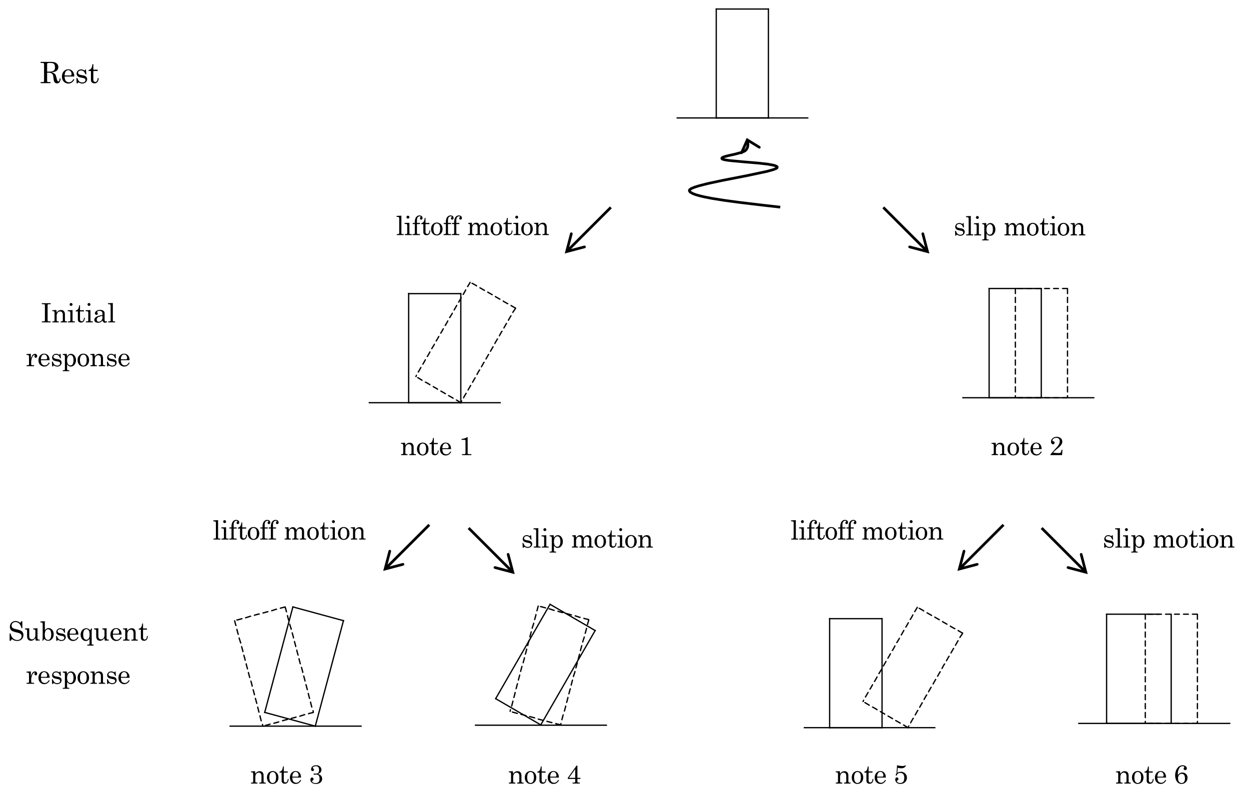


note 1 : equations of liftoff motion[*1]
liftoff commencement conditions

note 2 : equations of slip motion[*2]
slip commencement conditions

note 3 : equations of liftoff motion[*1]
liftoff termination conditions[*5]
overturning conditions[*6]

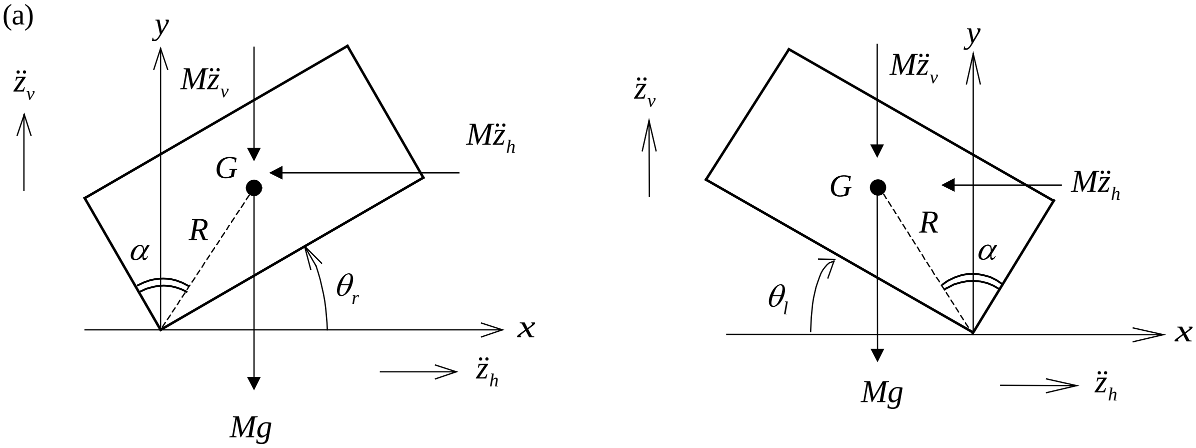
note 4 : equations of liftoff-slip interaction motion[*3]
slip commencement conditions including liftoff motion effects slip termination conditions[*4]
liftoff termination conditions[*5]
overturning conditions[*6]

note 5 : equations of liftoff-slip interaction motion[*3]
liftoff commencement conditions including slip motion effects
slip termination conditions[*4]
liftoff termination conditions[*5]
overturning conditions[*6]

note 6 : equations of slip motion[*2]
slip termination conditions[*4]

Figure 1. Classification of responses

(a)



[pivoting left bottom edge]

[pivoting right bottom edge]

(b)

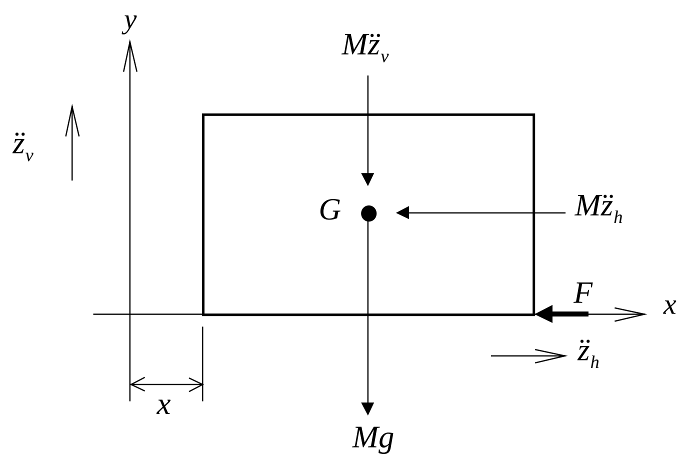


Figure 2. Initial responses [(a); liftoff, (b); Slip]

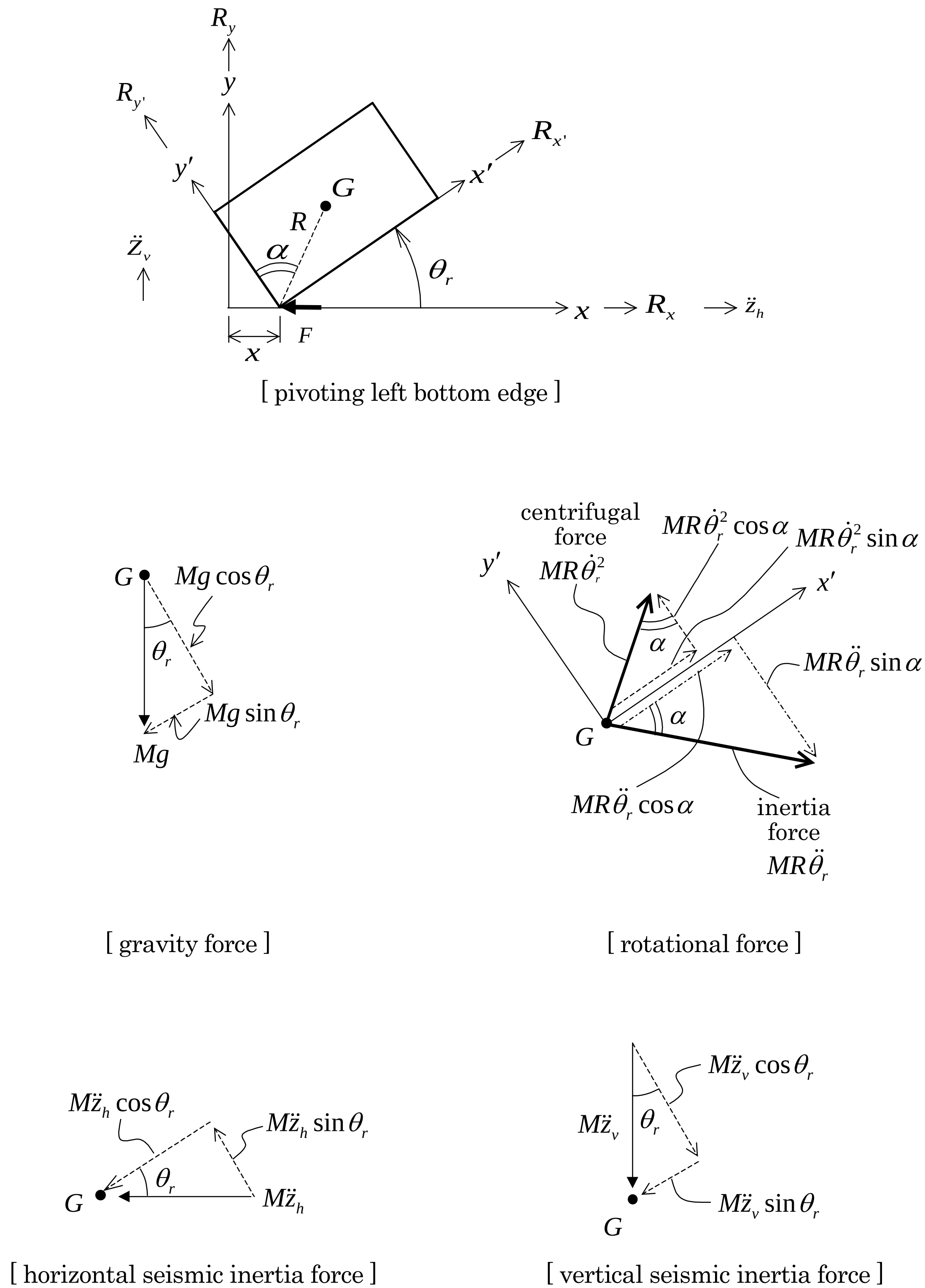


Figure 3. Equilibrium of forces during pivoting left bottom edge

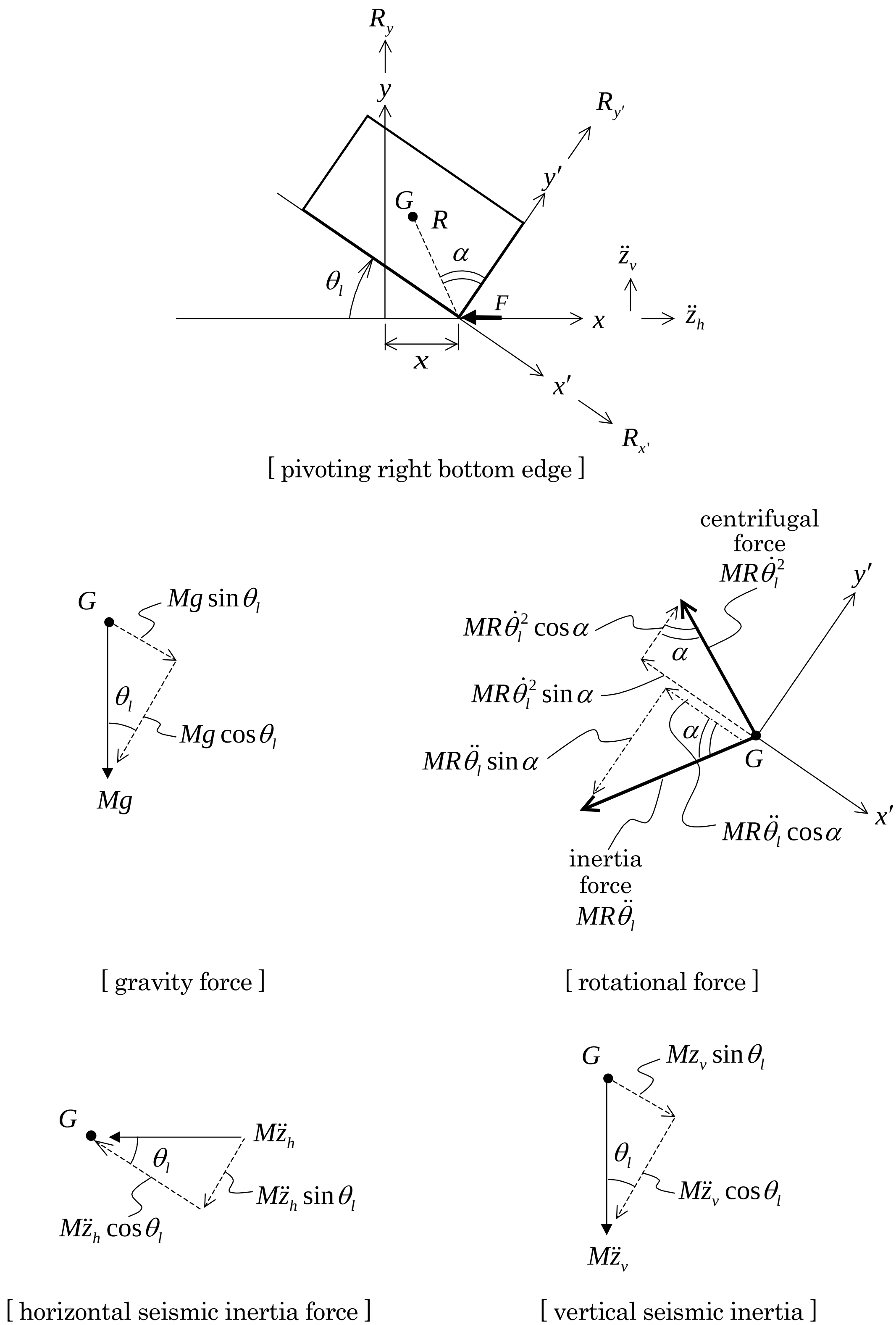


Figure 4. Equilibrium of forces during pivoting right bottom edge

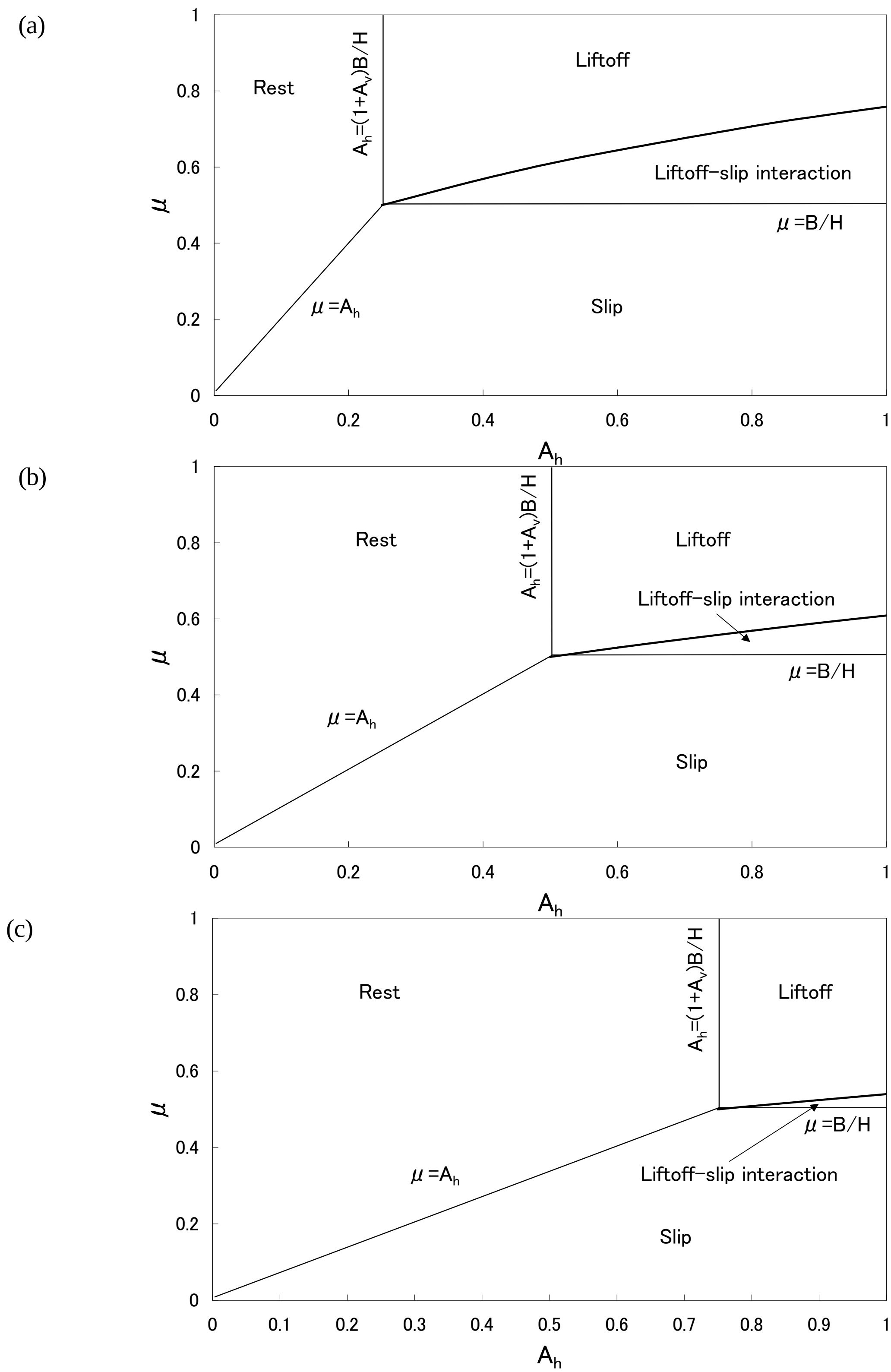
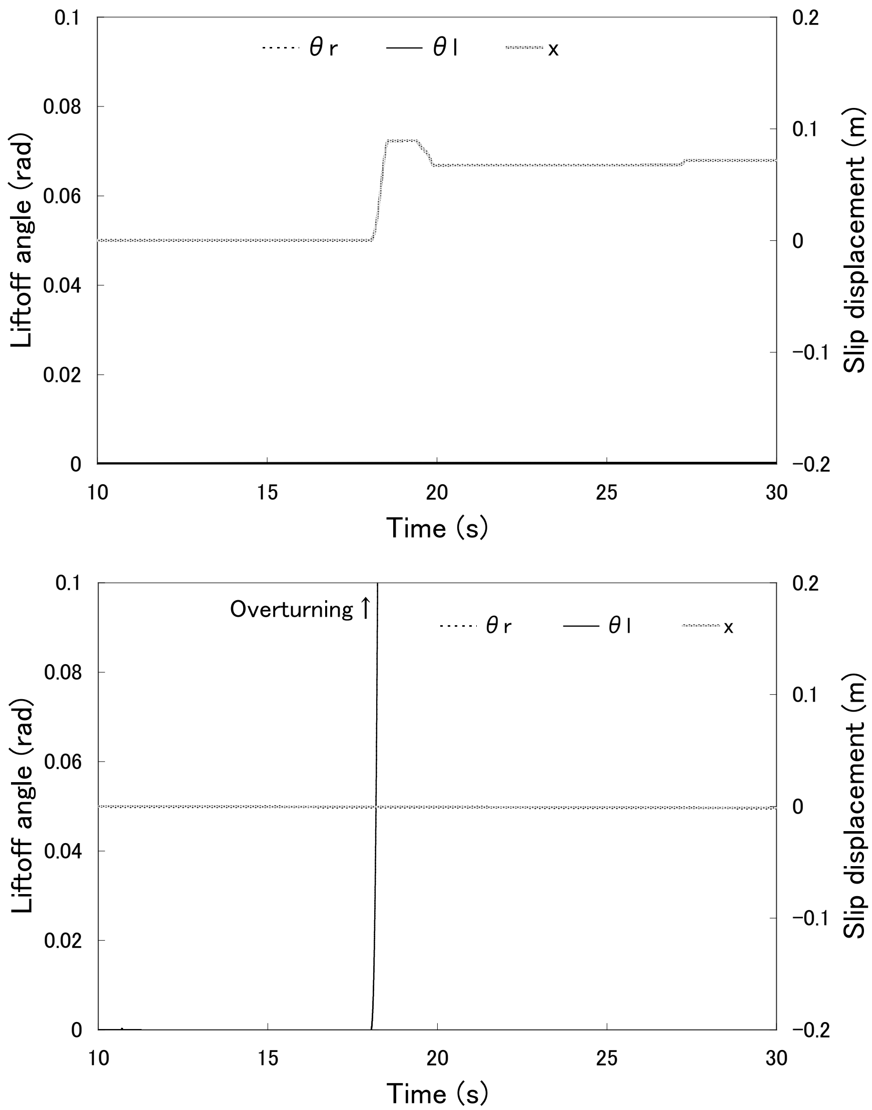


Figure 5. Criteria of 3 modes for B/H=2 [(a); A_v=-0.5, (b); A_v= 0.0, (c); A_v= 0.5]

(a)



(b)

Figure 6. [(a); μ =0.50, (b); μ =0.52 and 0.55]

(a)

(b)

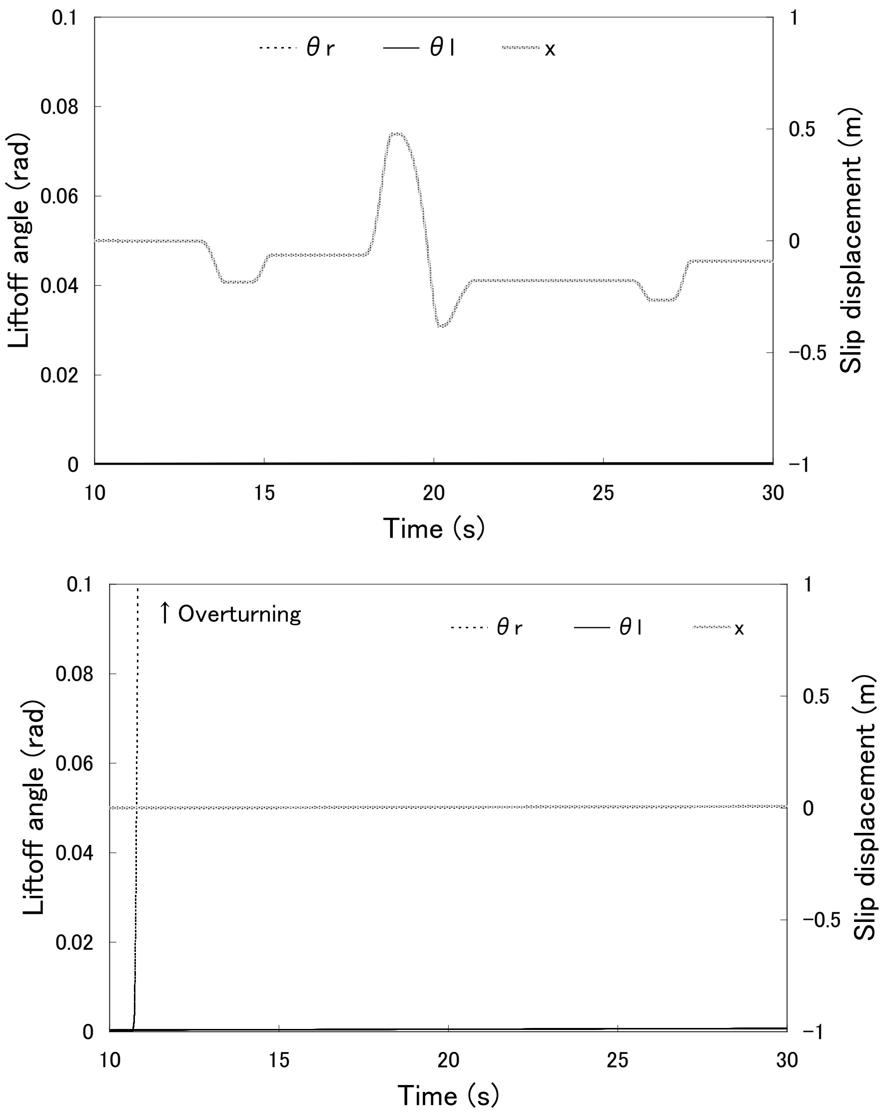


Figure 7. [(a); μ =0.50, (b); μ =0.52 and 0.55]

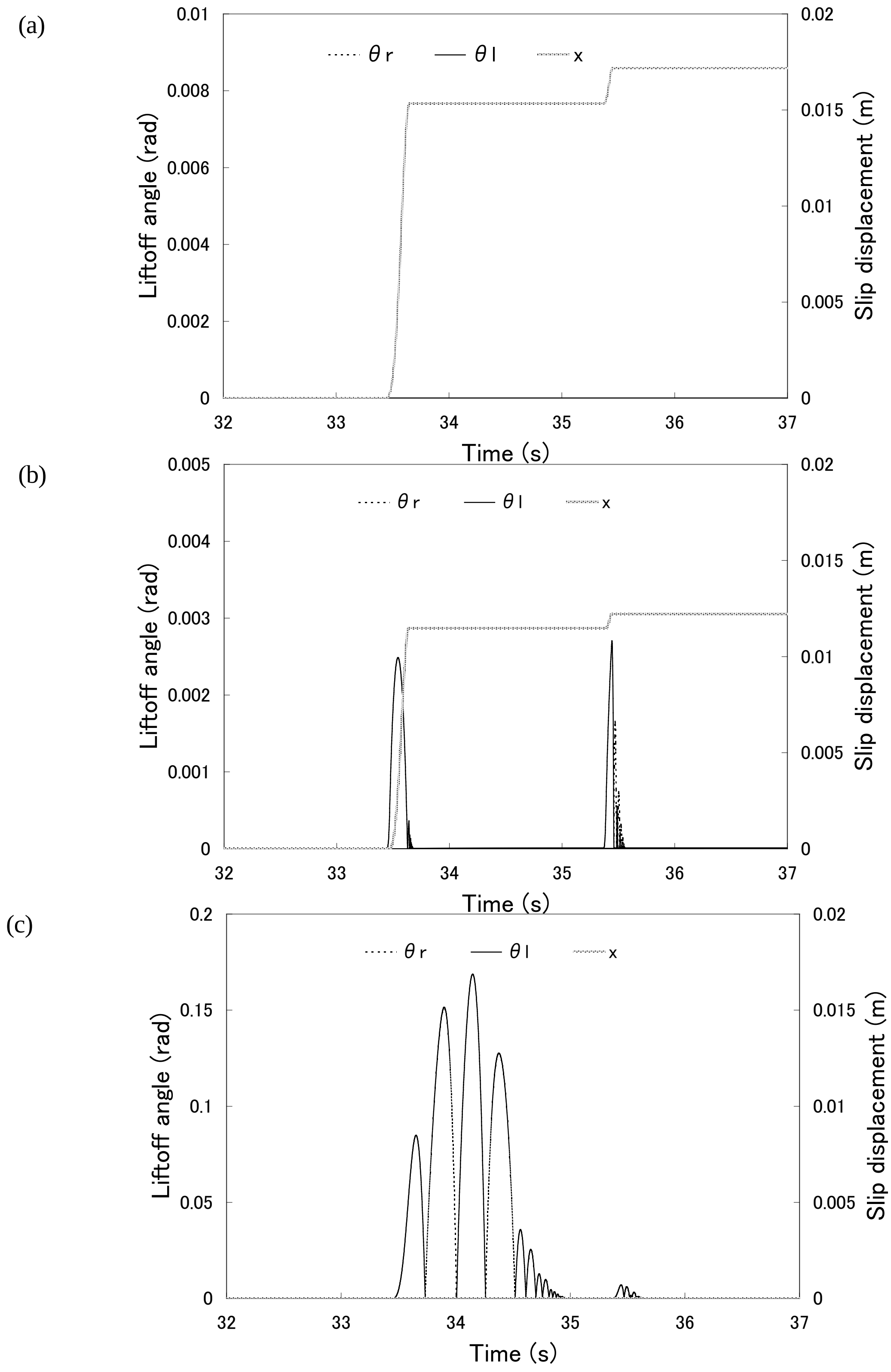


Figure 8. [(a); μ =0.50, (b); μ =0.52, (c); μ =0.55]

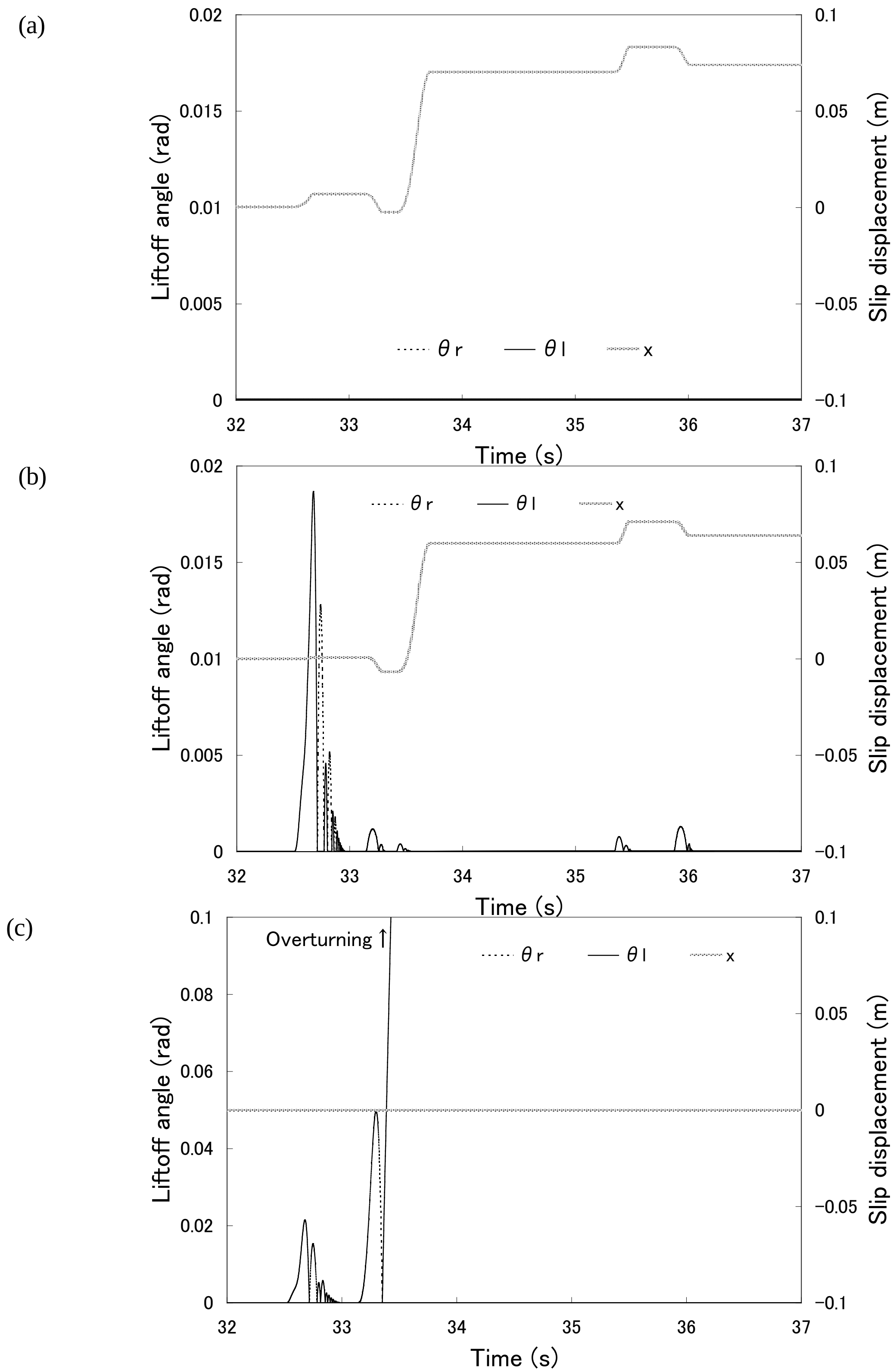


Figure 9. [(a); μ =0.50, (b); μ =0.52, (c); μ =0.55]

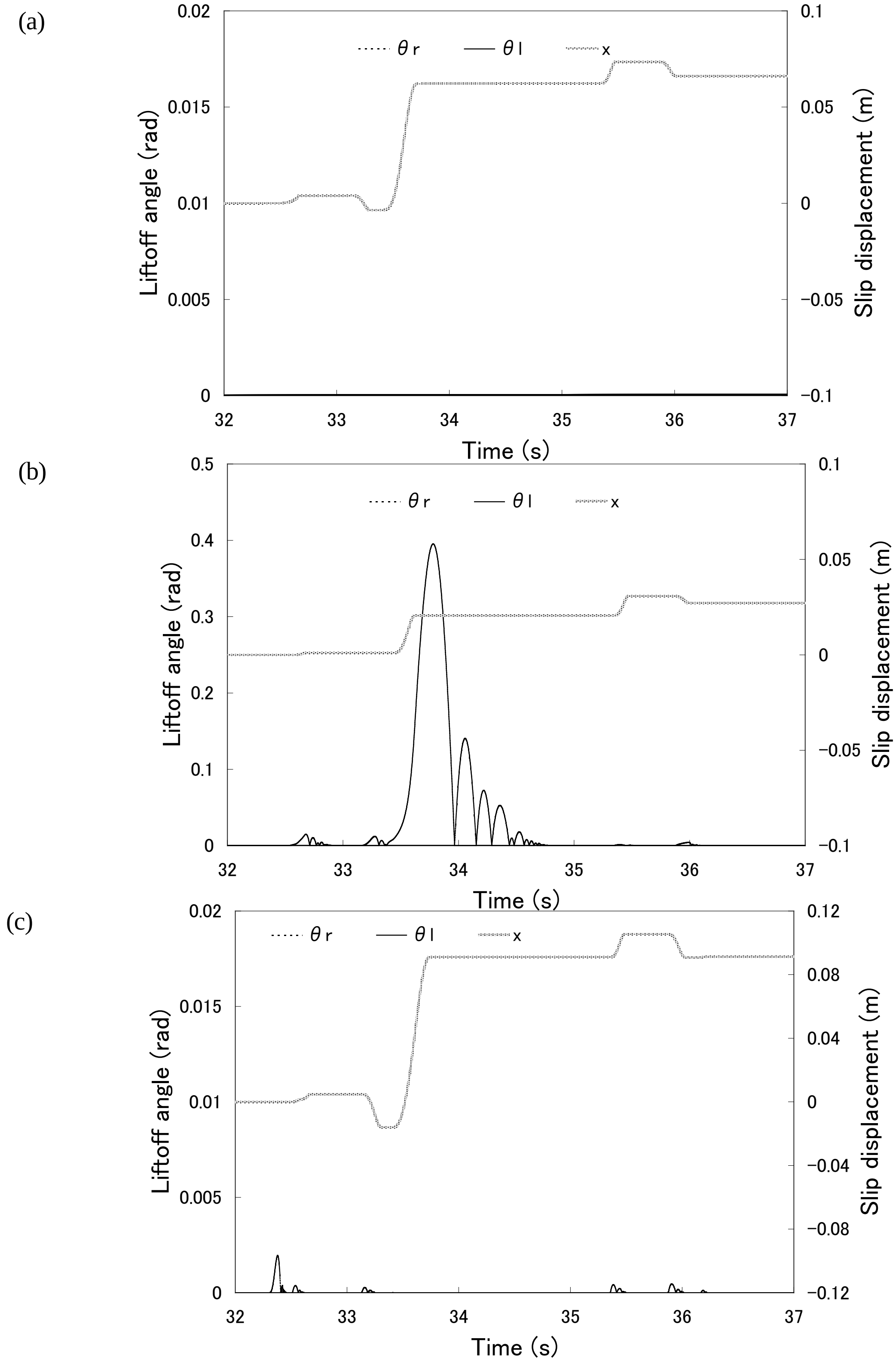


Figure 10. [(a); eliminating liftoff, (b); half vertical acc., (c); twice vertical acc.]

Figure No.	Name of earthquake	Max. horizontal acc.	Max. vertical acc.	Static friction coefficient	Judgement of Overturning	Descriptions
6(a)	Mexico	600.0 gal	172.8 gal	0.50	No	Slip only
6(b)	Mexico	600.0 gal	172.8 gal	0.52	Yes	No rebound and slip is observed
6(b)	Mexico	600.0 gal	172.8 gal	0.55	Yes	The same as above
7(a)	Mexico	800.0 gal	230.4 gal	0.50	No	Slip only
7(b)	Mexico	800.0 gal	230.4 gal	0.52	Yes	No rebound and slip is observed
7(b)	Mexico	800.0 gal	230.4 gal	0.55	Yes	The same as above
8(a)	Hyogoken-nanbu	600.0 gal	243.7 gal	0.50	No	Slip only
8(b)	Hyogoken-nanbu	600.0 gal	243.7 gal	0.52	No	Liftoff-Slip interaction is observed
8(c)	Hyogoken-nanbu	600.0 gal	243.7 gal	0.55	No	No slip is observed. The body avoids overturning
9(a)	Hyogoken-nanbu	800.0 gal	324.9 gal	0.50	No	Slip only
9(b)	Hyogoken-nanbu	800.0 gal	324.9 gal	0.52	No	Liftoff-slip interaction is observed
9(c)	Hyogoken-nanbu	800.0 gal	324.9 gal	0.55	Yes	No slip is observed
10(a)	Hyogoken-nanbu	800.0 gal	324.9 gal	0.52	No	Reference for Fig.9(b) with no liftoff motions
10(b)	Hyogoken-nanbu	800.0 gal	162.5 gal	0.52	No	Reference for Fig.9(b) with half vertical accelerations
10(c)	Hyogoken-nanbu	800.0 gal	649.8 gal	0.52	No	Reference for Fig.9(b) with twice vertical accelerations

Table 1. Analysis conditions and brief descriptions of events